STOCHASTIC GAMES ON GRAPHS

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Abstract

This thesis is about stochastic games and network games on graphs, more specifically infinite two-person games played on finite graphs with imperfect information. This thesis consists of two parts.

The first part treats determinacy (optimal value) and optimal (ϵ-optimal) strategies in stochastic games. The main concern of this part is to give simple expressions of values of games and investigate the existence of optimal (ϵ-optimal) strategies for each player in games of increasing Borel complexity, namely a reachability, Büchi game, and etc. Moreover, we are interested in the questions of what type of optimal (ϵ-optimal) strategy exists for each player depending on the type of games. We first prove some basic facts on a generalized version of reachability games, called generalized reachability games. We describe the value of generalized reachability games as a limit value of finite-step games. We also show that there exists a memoryless optimal strategy of Player II in any generalized reachability games, while Player I must settle for ϵ-optimality (memoryless). Intuitively, for a given finite play, memoryless strategies give the next action depending on the current state rather than the finite play itself. We next observe games with more complex objectives, games with Büchi objectives. The results of generalized reachability games are used to show the value of Büchi games can be approximated as values of some generalized reachability games. The proof includes the information on how we can construct ϵ-optimal strategies in a Büchi game for a given positive real number ϵ. But we see that, in general, Player I may not have a memoryless ϵ-optimal strategy in a Büchi game for some $\epsilon > 0$, and she is required the use of strategies with infinite memory indeed. However, we think that Player II has a finite memory ϵ-optimal strategy in any Büchi game. Informally, a finite memory strategy is a strategy which does not have to remember all the history of the play, and therefore is allowed to forget some information about the history of the play so far. In the final section of this part, we introduce games with a Boolean combination of reachability games. We first show that each player does not have a memoryless ϵ-optimal strategy in this game for some $\epsilon > 0$. We then prove that Player I and Player

II have finite memory ϵ-optimal strategies for any $\epsilon > 0$ in these games.

The second part is devoted to network games where our interests are the existence and effective computability of Nash equilibria with respect to graph properties. We model this game as a non-cooperative strategic game on a graph, and analyze the characteristics of Nash equilibria [30], which is a stable solution immune from unilateral deviation. In particular, we focus on showing the existence of mixed Nash equilibria corresponding to graph properties. We have considered various conditions for computing mixed Nash equilibria. Informally, we view a network game as a game on an undirected graph whose edges are exposed to virus infection disseminated by attackers, and nodes can be protected by defenders, for instance, system security software. In this model, attackers and defenders have opposing aims that seek to maximize damage and protection on networks, respectively. This work is continuous study of the network games introduced by Mavronicholas et al. [26] and also motivated from the study by MedSalem et al. [29]. We next generalize our network game to an asynchronous game, where two players repeatedly (unbounded many times) execute simultaneous games. The asynchronous game can be viewed as a special Blackwell game [3]. Martin proved that a Blackwell game with Borel payoff function is determinate, i.e., it has a stable solution [24]. Thus, our game with a suitable payoff is also determinate, and each player has a stable strategy. However, we show this by reducing it to a finite game without referring to Blackwell determinacy.