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Stability analysis of a simple mathematical model for unemployment

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Unemployment is a highly concerned problem due to its impact on social, financial and political stability of a nation. The problem is complicated by the continual increase of labour forces inclusion combined with limited availability of jobs. In this paper, a simple mathematical model for unemployment is proposed by considering two variables: the number of employed persons and the number of unemployed persons. The assumptions employed to develop the mathematical model are discussed. It is shown that the model has only one non-negative equilibrium. Stability analysis conducted using the Routh-Hurwitz criterion imply that the model has a locally asymptotically stable equilibrium point.

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1. Introduction

Unemployment has been a concern to most nations as a threatening crisis due to its impact in social and political stability. The continuous increment of unemployment rate is caused by various factors and aspects such as economic declining, increases of population and technology advancement. Poverty and unstable income among the citizen which can lead to the increases in crimes and family disunity are some examples resulting from unemployment.

Population rise, economic decline and technology advancement are major factors associated with unemployment rate increment (Misra and Singh, 2011). Annual rise of graduates and labour forces in tandem with limited job availability results in greater competition for jobs, causing unemployment of less qualified candidates. On the other hand, there a groups of unemployed persons who lose their job when the companies go through retrenchment or bankruptcy. Unemployment is also an adverse effect brought forward by advancement of technology that is expected to improve the service quality and minimise the production time and cost.

In 1982, the international definitions and criteria of employment and unemployment was presented in the Thirteenth International Conference of Labour Statistician. It was stated that (Hunssmanns et al., 1990)

"The "unemployed" comprise all persons above a specified age who during the reference period were: (1)"without work", i.e. were not in paid employment or self-employment, as specified by the international definition of employment; (2)"currently available for work", i.e. were available for paid employment or self-employment during the reference period; and (3)"seeking work", i.e. had taken specific steps in a specified reference period to seek paid employment or self-employment."

The paper is organized as follows. Section 2 discusses the variables and parameters in the models and the assumptions that have been employed to govern the mathematical model.

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Section 3 discusses the existence of the equilibrium and the stability analysis of the equilibrium.

2. Methodology

For the mathematical modeling of unemployment, the main variables considered in this paper are the numbers of employed persons and unemployed person, denoted by U and E, respectively at any time, t. In the modeling process, some assumptions are required. It is assumed that some of the unemployed persons may directly get employment and some of the employed persons may join the unemployment class (i.e. fired or resign). All entrants to the unemployment class are assumed to be fully qualified to be employed (i.e. migrants, graduates or postgraduates). It is also assumed that the unemployed persons are continuously increasing in number and are migrating from their living place to other places to obtain employment. The total number of available vacancies are limited and assumed to be constant.

The modeling of the unemployment can be summarized in Figure 1:

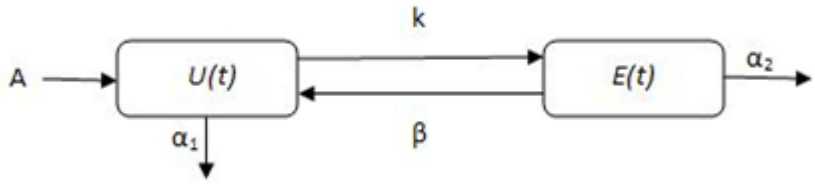

Figure 1: A compartmental diagram of the unemployment model.

where U is the number of unemployed persons, E is the number of employed persons, A is a positive constant rate of entrants to the unemployment class, k is a positive constant of proportionality which represents the rate of change of the unemployed person to employed person, β is a positive constant which represents the rate of employed persons joining the unemployment class (i.e. fired or resign), α_1 is a positive constant which represents the rate of migration or death of unemployed person, α_2 is a positive constant which represents the rate of retirement or death of employed person.

From the diagram, further assumptions are made in order to formulate the differential equations for the rate of change of the number of unemployed persons U and the rate of change of the number of employed persons. The rate of migration or death of unemployed persons, α_1 is assumed to be proportional to the number of unemployed persons, U. The rate of retirement or death of employed persons, α_2 is assumed to be proportional to the number of employed persons, E. The rate of employed persons joining the unemployment class (i.e. fired or resign), β is assumed to be proportional to the number of employed persons, E. The rate of change of the number of unemployed persons gaining employment, k will be jointly proportional to U and to the number of available vacancies ($E_a - E$).

Thus, the rates of change of the number of unemployed persons, dU/dt and the rates of change of the numbers of employed persons, dE/dt are represented by the equations:

$$\frac{dU}{dt} = A - kU(E_a - E) + \beta E - \alpha_1 U \qquad (1)$$

$$\frac{dE}{dt} = kU(E_a - E) - \beta E - \alpha_2 E \qquad (2)$$

where E_a is the total number of available vacancies. The proposed model for unemployment is governed by Eqs. (1) and (2).

3. Results and Discussion

3.1. Equilibrium analysis

The model governed by Eqs. (1) and (2) has only one non-negative equilibrium, $Q(U^*, E^*)$, which may be obtained by solving Eqs. (1) and (2) when they are equal to zero. The resulting equations are

$$E = \frac{k \cdot U \cdot E_a}{(k \cdot U + \beta + \alpha_2)} \qquad (3)$$

and

$$A - \frac{k \cdot U \cdot E_a \cdot \alpha_2}{k \cdot U + \beta + \alpha_2} - \alpha_1 \cdot U = 0\,. \qquad (4)$$

Now, to show the existence of equilibrium, we solve Eq. (4) to get the equilibrium U^*:

$$U = \frac{1}{2\alpha_1 k}\left(\left(Ak - kE_a\alpha_2 - \alpha_1\beta - \alpha_1\alpha_2\right) \pm D\right)$$

where

$$D = \sqrt{\begin{array}{c} A^2k^2 - 2Ak^2E_a\alpha_2 + 2Ak\alpha_1\beta + 2Ak\alpha_1\alpha_2 \\ + k^2E_a^2\alpha_2^2 + 2kE_a\alpha_1\alpha_2\beta + 2kE_a\alpha_1\alpha_2^2 \\ + \alpha_1^2\beta^2 + 2\alpha_1^2\alpha_2\beta + \alpha_1^2\alpha_2^2 \end{array}}.$$

We know that the constant rate of entrants to the unemployment class is greater than the number of available vacancies, $A > E_a$, giving $D > 0$ and $Ak - kE_a\alpha_2 - \alpha_1\beta - \alpha_1\alpha_2 > 0$ Therefore, we get:

$$U_1 = \frac{1}{2\alpha_1 k}\left(\left(Ak - kE_a\alpha_2 - \alpha_1\beta - \alpha_1\alpha_2\right) + D\right)$$

and

$$U_2 = \frac{1}{2\alpha_1 k}\left(\left(Ak - kE_a\alpha_2 - \alpha_1\beta - \alpha_1\alpha_2\right) - D\right)$$

which is the positive and negative roots. Since we are dealing with the number of unemployed person, U, we neglected the negative value for U^* and leave us with:

$$U^* = \frac{1}{2\alpha_1 k}\left(\left(Ak - kE_a\alpha_2 - \alpha_1\beta - \alpha_1\alpha_2\right) + D\right). \quad (5)$$

Then, substituting Eq. (5) into Eq. (3), we get a positive value of E^*.

$$E^* = \frac{k \cdot U^* \cdot E_a}{(k \cdot U^* + \beta + \alpha_2)}. \quad (6)$$

Hence, the model (1) and (2) has only one non-negative equilibrium, $Q(U^*, E^*)$.

3.2. Stability analysis

A stability analysis is conducted to examine the local stability of the equilibrium for the model. In order to investigate the local stability behavior of the equilibrium, we construct a Jacobian matrix for the system and analyze the characteristic equation obtained based on the Routh-Hurwitz criterion.

In order to investigate the local stability behaviour of equilibrium $Q(U^*, E^*)$, we compute the following Jacobian Matrix for the Eqs. (1) and (2):

$$J = \begin{bmatrix} -k\left(E_a - E\right) - \alpha_1 & kU + \beta \\ k\left(E_a - E\right) & -kU - \beta - \alpha_2 \end{bmatrix}.$$

Next, evaluating the Jacobian corresponding to the equilibrium $Q(U^*, E^*)$, and finding the characteristic equation, we obtain

$$\lambda^2 + a_1\lambda + a_2 = 0 \quad (7)$$

where

$$a_1 = kE_a - kE^* + \alpha_1 + kU^* + \beta + \alpha_2,$$
$$a_2 = k\left(E_a\alpha_2 - E^*\alpha_2 + U^*\alpha_1\right) + \alpha_1\left(\beta + \alpha_2\right).$$

From simple algebraic manipulation, we know that $E^* < E_a$ and it is clear that a_1 and a_2 are positive. Therefore, $a_1a_2 > 0$. Now, by using Routh-Hurwitz criterion (Hunssmanns et al., 1990) we can say that the roots of Eq. (7) are either negative or have a negative real part. Hence the model is said to have a locally asymptotically stable equilibrium point $Q(U^*, E^*)$.

4. Conclusion

In this paper, a two dimensional mathematical model is proposed and analyzed in order to understand the problem of unemployment. The variables used are the number of unemployed and employed persons. Some assumptions such as the entrants of fully qualified employment candidates and the limited vacancies are considered as constant are required to formulate the model. It is shown that the developed model has one non-negative, locally asymptotically stable equilibrium.

The model of unemployment, generally, can still be further improved. For example, more variables can be taken into consideration such as the numbers of temporary employed person (Misra and Singh, 2011) and other factors that can affect the unemployment rate such inflation and salary (Kooros, 2006). Finally, the study can still be further improved by conducting some numerical simulation using MATLAB or MAPLE to check the feasibility of the stability analysis.

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