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Log-aesthetic flow governed by heat conduction equations

Kenjiro T. Miura [a], Sho Suzuki [a], R. U. Gobithaasan [b], Péter Salvi [c] and Shin Usuki [a]

[a]Shizuoka University, Japan; [b]Universiti Malaysia Terengganu, Malaysia; [c]Budapest University of Technology and Economics, Hungary

ABSTRACT
In this research, based on the concept of the smoothing by curve shortening flow and curvature flow, we will propose log-aesthetic flow to make free-form curves "log-aesthetic." We will discuss smoothing methods that deal with continuous curves as well as discrete ones. We have one degree of freedom α to control smoothing for log-aesthetic flow and can expect the completely smoothed shape of a given curve, which means the shape obtained by fairing, to be log-aesthetic curve. This paper shows that log-aesthetic flow is basically governed by the heat conduction equation, which has been well studied in both physics and mechanical engineering, its effects are easily inferred by the designers and it will be useful for practical aesthetic design.

KEYWORDS
Log-aesthetic Flow; Log-aesthetic Curve; Heat Conduction Equation; Variational Principle

1. Introduction

Geometric flows such as curve shortening and curvature flows are used for various fields including CAD [10,11], CG [1,2] and Computer Vision [6,7]. The curve shortening flow successively decreases the total length of a given curve as its name indicates and the curvature flow produces gradually smoother approximations of a given curve by reducing a fairing energy. Recently for curvature flow Crane et al. [1] proposed a very stable and very fast method to smooth discrete curves and surfaces by changing variables from vertices' positions to their curvatures for reducing fairing energies. They achieved extraordinary stability even on highly degenerate meshes and integration time steps orders of magnitude larger than existing methods.

Log-aesthetic curves include the logarithmic (equiangular) spiral, clothoid, and involute curves. Although most of these are expressed only by an integral form of the tangent vector, it is possible to interactively generate and deform them, and they are expected to be utilized in industrial and graphical design. We think that to make a curve drawn by a designer aesthetically attractive is an important research issue because sketches or hand drawings are still a very natural interface for the designer to transfer his/her ideas on shapes to computers. In this research, based on the concept of the smoothing by curve shortening flow and curvature flow, we will propose log-aesthetic flow to make free-form curves "log-aesthetic." We will discuss smoothing methods that deal with continuous curves as well as discrete ones.

The rest of the paper is organized as follows. Section 2 reviews the basics of curvature shortening and curvature flows. Section 3 introduces the main topic of this paper; log-aesthetic (LA) flow. Sections 4 and 5 discusses the continuous and discrete types of LA flow, respectively. Section 6 concludes the paper with a discussion on future work.

2. Related work

In this section, we discuss related researches on the log-aesthetic curve, curve shortening and curvature flows [5].

2.1. The log-aesthetic curve

"Aesthetic curves" were proposed by Harada [5] as such curves whose logarithmic distribution diagram of curvature (LDDC) is approximated by a straight line. Miura [8] derived analytical solutions of the curves whose logarithmic curvature graph (LCG): an analytical version of the LDDC [5] are strictly given by a straight line and proposed these lines as general equations of aesthetic curves. Furthermore, Yoshida and Saito [12] analyzed the properties of the curves expressed by the general equations and developed a new method to interactively generate a curve by specifying two end points and the tangent

CONTACT Kenjiro T. Miura tmkmiur@ipc.shizuoka.ac.jp

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vectors there with three control points as well as α: the slope of the straight line of the LCG. In this research, we call the curves expressed by the general equations of aesthetic curves the log-aesthetic curves.

2.1.1. General equations of aesthetic curves

For a given curve, we assume the arc length of the curve and the radius of curvature are denoted by s and ρ, respectively. The horizontal axis of the logarithmic curvature graph measures $\log \rho$ and the vertical axis measures $\log |ds/d\log\rho| = \log|\rho ds/d\rho|$. If the LCG is given by a straight line, there exists a constant α such that the following equation is satisfied:

$$\log\left|\rho\frac{ds}{d\rho}\right| = \alpha\log\rho + C \tag{2.1}$$

where C is a constant. The above equation is called the fundamental equation of aesthetic curves [8]. Rewriting Eqn. (2.1), we obtain:

$$\frac{1}{\rho^{\alpha-1}}\frac{ds}{d\rho} = e^C = C_0 \tag{2.2}$$

Hence, there is some constant c_0 such that:

$$\rho^{\alpha-1}\frac{d\rho}{ds} = c_0 \tag{2.3}$$

From the above equation, when $\alpha \neq 0$, the first general equation of aesthetic curves

$$\rho^\alpha = c_0 s + c_1 \tag{2.4}$$

is obtained. If $\alpha = 0$, we obtain the second general equation of aesthetic curves aesthetic curves

$$\rho = c_0 e^{c_1 s} \tag{2.5}$$

A curve that satisfies Eqn. (2.4) or Eqn. (2.5) is called a log-aesthetic curve.

2.1.2. Parametric expressions log-aesthetic curves

In this subsection, we will show parametric expressions of the log-aesthetic curves.

We assume that a curve $C(s)$ satisfies Eqn. (2.4). Then

$$\rho(s) = (c_0 s + c_1)^{\frac{1}{\alpha}} \tag{2.6}$$

As S is the arc length, $|dC(s)/ds| = 1$ (for example, refer to [5]) and there exists $\theta(s)$ satisfying the following two equations:

$$\frac{dx}{ds} = \cos\theta, \quad \frac{dy}{ds} = \sin\theta \tag{2.7}$$

Since $\rho = d\theta/ds$,

$$\frac{d\theta}{ds} = (c_0 s + c_1)^{-\frac{1}{\alpha}} \tag{2.8}$$

If $\alpha \neq 1$,

$$\theta = \frac{\alpha(c_0 s + c_1)^{\frac{\alpha-1}{\alpha}}}{(\alpha-1)c_0} + c_2 \tag{2.9}$$

If the start point of the curve is given by $P_0 = C(0)$,

$$C(s) = P_0 + e^{ic_2}\int_0^s e^{i\frac{\alpha(c_0 u + c_1)^{\frac{\alpha-1}{\alpha}}}{(\alpha-1)c_0}}\,du \tag{2.10}$$

For the second general equation of aesthetic curves expressed by Eqn. (2.9),

$$\frac{d\theta}{ds} = \frac{1}{c_0}e^{-c_1 s} \tag{2.11}$$

$$\theta = -\frac{1}{c_0 c_1}e^{-c_1 s} + c_2 \tag{2.12}$$

Therefore the curve is given by

$$C(s) = P_0 + e^{ic_2}\int_0^s e^{-\frac{1}{c_0 c_1}e^{-c_1 u}}\,du = \pi r^2 \tag{2.13}$$

2.2. Curve shortening flow and curvature flow

We deal with a curve $\mathbf{C}(p, t)$ defined by parameter p $(0 \leq p \leq 1)$ which deforms with time t. We assume that that its total length is a function of t and express it as $L(t)$. Then

$$L(t) = \int_0^1 \left\|\frac{\partial \mathbf{C}}{\partial p}\right\| dp \tag{2.14}$$

where $\|\boldsymbol{v}\|$ means the norm of vector $\boldsymbol{v}$. By differentiating the above equation with respect to t, we obtain

$$L'(t) = \int_0^1 \frac{\left\langle \frac{\partial \mathbf{C}}{\partial p}, \frac{\partial^2 \mathbf{C}}{\partial p \partial t}\right\rangle}{\left\|\frac{\partial \mathbf{C}}{\partial p}\right\|} dp \tag{2.15}$$

where $\boldsymbol{a}, \boldsymbol{b}$ means the inner product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. By performing partial integration to Eqn. (2.15),

$$L'(t) = \left[\frac{\left\langle \frac{\partial \mathbf{C}}{\partial p}, \frac{\partial \mathbf{C}}{\partial t}\right\rangle}{\left\|\frac{\partial \mathbf{C}}{\partial p}\right\|}\right]_0^1 - \int_0^1 \left\langle \frac{\partial \mathbf{C}}{\partial t}, \frac{\partial}{\partial p}\left[\frac{\frac{\partial \mathbf{C}}{\partial p}}{\left\|\frac{\partial \mathbf{C}}{\partial p}\right\|}\right]\right\rangle dp \tag{2.16}$$

We assume that both of the end point positions of the curve are fixed with respect to time, i.e. $\partial\mathbf{C}(0, t)/\partial t =$

$\partial \boldsymbol{C}(1, t)/\partial t = 0$. Then

$$L^{'(t)} = -\int_0^{L(t)} \left\langle \frac{\partial \boldsymbol{C}}{\partial t}, \kappa N \right\rangle ds \tag{2.17}$$

where s is an arc length and it is given by $ds = \|\partial C/\partial p\| dp$. κ and $\boldsymbol{N}$ are the curvature and the normal vector, respectively and they are defined by $\kappa \boldsymbol{N} = \partial^2 \boldsymbol{C}/\partial s^2$. Hence when

$$\frac{\partial \boldsymbol{C}}{\partial t} = \kappa N, \tag{2.18}$$

then $L(t)$ will decrease the most quickly. This flow is called *curve shortening flow*.

Here we define the curve's energy as $E(t) = \int_0^{L(t)} \kappa^2 ds$. Then

$$E'(t) = 2\int_0^{L(t)} \kappa \frac{\partial \kappa}{\partial t} ds, \tag{2.19}$$

Therefore E(t) will decrease the most rapidly when $\partial \kappa/\partial t = -2\kappa$. This flow also deforms the shape of the curve using curvature and it is called *curvature flow*.

3. Log-aesthetic flow

In this section we propose two types of log-aesthetic flow: *length-based* and *energy-based*.

3.1. Arc-length functional of the log-aesthetic curve in aesthetic space

The functional of the log-aesthetic curve which satisfies $\sigma = \rho^\alpha = cs + d$ is given by the following expression [9]:

$$J(t) = \int_0^L \sqrt{1 + \sigma_s^2} ds. \tag{3.1}$$

Hence

$$J'(t) = \int_0^L (1 + \sigma_s^2)^{-\frac{1}{2}} \sigma_s \sigma_{st} ds = \left[(1 + \sigma_s^2)^{-\frac{1}{2}} \sigma_s \sigma_t \right]_0^L - \int_0^L \frac{\sigma_{ss}}{(1 + \sigma_s^2)^{\frac{3}{2}}} \sigma_t ds \tag{3.2}$$

We assume that both of the curvatures at the end points are fixed with respect to time, i.e. $\partial \sigma(0, t)/\partial t = \partial \sigma(L, t)/\partial t = 0$. Then

$$J'(t) = -\int_0^L \frac{\sigma_{ss}}{(1 + \sigma_s^2)^{\frac{3}{2}}} \sigma_t ds \tag{3.3}$$

Then J(t) will decrease the most rapidly when

$$\sigma_t = \frac{\sigma_{ss}}{(1 + \sigma_s^2)^{\frac{3}{2}}} \tag{3.4}$$

In this paper we call this type of the flow *length-based log-aesthetic flow*.

3.2. Energy functional of the log-aesthetic curve in aesthetic space

It is known that the problem to minimize the length of a curve is equivalent to that to minimize its energy [3]. The energy of the log-aesthetic curve corresponding to Eqn. (3.1) [9] is given by

$$J_E(t) = \frac{1}{2} \int_0^L (1 + \sigma_s^2) ds. \tag{3.5}$$

From the above equation and the assumption that $\partial \sigma(0, t)/\partial t = \partial \sigma(L, t)/\partial t = 0$, we obtain

$$J_E'(t) = -\int_0^L \sigma_{ss} \sigma_t ds. \tag{3.6}$$

Therefore J_E(t) will decrease the most rapidly when

$$\sigma_t = \sigma_{ss}. \tag{3.7}$$

The above equation approximates Eqn. (3.4) and we call this type of the flow *energy-based log-aesthetic flow*.

4. Continuous log-aesthetic flow

The heat conduction equation in one dimension is generally given by the following equations [4]:

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial s^2}, \quad 0 < s < L, \quad 0 < t \tag{4.1}$$

where u is an unknown function representing temperature and s is a parameter representing position. $a > 0$ and a is called thermal conductivity. L is the total length of the object to be analyzed. Hence Eqn. (4.1) can be interpreted as a heat conduction equation with $a = 1$ on $\sigma = \rho^\alpha = \kappa^{-\alpha}$ of a curve and we can describe the change of curvature, i.e. the deformation of the curve as the temperature change by heat conduction. This fact means that the energy-based log-aesthetic flow basically behaves as heat conduction and by solving the heat conduction equations we can obtain the shape of the curve deformed continuously by log-aesthetic flow. We will discuss how to solve Eqn. (4.1) under various conditions below.

4.1. In case where $\kappa = 0$ at both of the end points

Here we assume that $\alpha = -1$ and Eqn. (4.1) becomes

$$\kappa_t = \kappa_{ss}. \tag{4.2}$$

Furthermore we assume that the curvatures at both of the end points of the curve are equal to 0 and the total length of the curve is L. The initial and boundary conditions are given by $\kappa(s,\ 0) = f(s)$ and $\kappa(0,\ t) = \kappa(L,\ t) = 0$, respectively. We assume that a solution $\kappa(s,\ t)$ is the product of $S(s)$ of only parameter s and $T(t)$ of only parameter t as

$$\kappa(s,\ t) = S(s)T(t). \tag{4.3}$$

From the above discussion and the principle of superposition, the general solution $\kappa(s,\ 0)$ is given by

$$\kappa(s,\ t) = \sum_{m=1}^{\infty} a_m \sin\left(\frac{m\pi}{L}s\right) \exp\left(-\frac{m^2\pi^2}{L^2}t\right) \tag{4.4}$$

Therefore a_m is given by

$$a_m = 2\int_0^L \sin\left(\frac{m\pi}{L}s\right) f(s)ds, \quad m = 1,\ 2,\ \cdots \tag{4.5}$$

For example, when $L = 1$ and $f(s) = \sin(\pi s)$, from Eqn. (4.5)

$$a_m = \begin{cases} 1, & m = 0 \\ 0, & m > 1 \end{cases} \tag{4.6}$$

and

$$\kappa(s,\ t) = \sin(\pi s)\exp(-\pi^2 t) \tag{4.7}$$

Hence as t approaches infinity, $\kappa(s) \to 0$ and the curve converges to a straight line.

4.2. In case where $\kappa \neq 0$ at the end points: inhomogeneous boundary conditions

Again we assume that $\alpha = -1$ and the curvatures of the end points of the curves are fixed κ_0 and κ_1, respectively. We define a function $\gamma(s,\ t)$ whose curvatures of the end points are equal to be 0 as follows:

$$\gamma(s,\ t) = \kappa(s,\ t) - \frac{1}{L}(\kappa_0(L-s) + \kappa_1 s) \tag{4.8}$$

Using Eqn. (4.4), the following general equation is obtained:

$$\gamma(s,\ t) = \sum_{m=1}^{\infty} a_m \sin\left(\frac{m\pi}{L}s\right) \exp\left(-\frac{m^2\pi^2}{L^2}t\right) \tag{4.9}$$

Therefore

$$\begin{aligned}\kappa(s,\ t) &= \gamma(s,\ t) + \frac{1}{L}(\kappa_0(L-s) + \kappa_1 s) \\ &= \sum_{m=1}^{\infty} a_m \sin\left(\frac{m\pi}{L}s\right) \exp\left(-\frac{m^2\pi^2}{L^2}t\right) \\ &\quad + \frac{1}{L}(\kappa_0(L-s) + \kappa_1 s)\end{aligned} \tag{4.10}$$

As t approaches infinity, $\kappa(s) \to (\kappa_0(L-s) + \kappa_1 s)/L$, the curvature of the curve is given by a linear function of s and the curve converges to a clothoid curve. For example, when $L = 1$, $f(s) = \sin(\pi s)$, $\kappa_0 = 1$, and $\kappa_1 = 0$, from Eqn. (4.4),

$$a_m = \begin{cases} 1, & m = 0 \\ 0, & m > 1 \end{cases} \tag{4.11}$$

and

$$\kappa(s,\ t) = \sin(\pi s)\exp(-\pi^2 t) + 1 - s \tag{4.12}$$

Fig. 1. shows curvature distributions and the shapes of the curves deformed by log-aesthetic flow. Hence as t approaches infinity, $\kappa(s) \to 0$ and the curve converges to a straight line. The right figure shows curves that are modified to pass through the given end points of the initial curve by iterative processes by shortening the curve gradually to satisfy the given curvatures. During the iterative processes, the curve must be rotated about the start point to satisfy the end point condition. If the distance between the start and end points of the curve is shorter than the specified distance, the length of the curve is increased. Otherwise it is decreased. Our method can generate a curve between given two points and two end curvatures.

4.3. Closed curve case

Here we also assume that $\alpha = -1$. We define a function $\gamma(s,\ t)$ whose curvatures of the start and end points are always equal as follows:

$$\kappa(0,\ t) = \kappa(L,\ t) \tag{4.13}$$

since the curve is closed. In the above equation L is the total length of the curve. The initial condition is given by

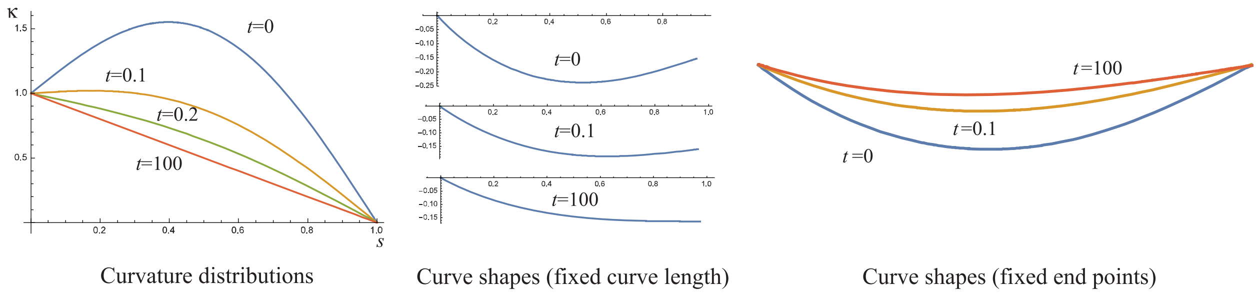


Figure 1. Curvature distributions and their curve shapes deformed by log-aesthetic flow.

$\kappa(s,\ 0) = f(s)$ assuming that $f(0) = f(L)$. The following general equation is obtained:

$$\kappa(s,\ t) = \sum_{m=1}^{\infty} \left(a_m \sin\left(\frac{m\pi}{L}s\right) + b_m \cos\left(\frac{m\pi}{L}s\right)\right) \times \exp\left(-\frac{m^2\pi^2}{L^2}t\right) \tag{4.14}$$

where

$$a_m = 2\int_0^L \sin\left(\frac{m\pi}{L}s\right) f(s)ds,$$

$$b_m = 2\int_0^L \cos\left(\frac{m\pi}{L}s\right) f(s)ds, \quad m = 0,\ 1,\ 2,\ \cdots \tag{4.15}$$

As t approaches infinity, the curvature of the curve becomes constant everywhere and the curve converges to a circle whose radius is the average value of $f(s)$.

4.4. General α case

We deal with the following problem: Assume that the total length of the curve is L and the initial condition is $\kappa(s,\ 0) = f(s)$. Furthermore the curvatures of the end points of the curves are fixed κ_0 and κ_1, respectively. Then

$$\frac{\partial \sigma}{\partial t} = \frac{\partial^2 \sigma}{\partial s^2}, \quad 0 < s < L, 0 < t, \quad \sigma(s, 0) = f(s)^{-\alpha},$$
$$\sigma(0, t) = \kappa_0^{-\alpha}, \sigma(L, t) = \kappa_1^{-\alpha}. \tag{4.16}$$

By solving the above partial differential equation, $\sigma(s,\ t)$ is obtained and $\kappa(s,\ t) = \ \sigma(s,\ t)^{-\frac{1}{\alpha}}$ is determined. For example, when $\alpha = -1/2, L = 1,\ f(s) = \sin^2(\pi s), \kappa_0 = 1$, and $\kappa_1 = 0$, from Eqn. (4.12),

$$\sigma(s,\ t) = \sin(\pi s)\exp(-\pi^2 t) + 1 - s \tag{4.17}$$

So the curvature $\kappa(s,\ t)$ is given by

$$\kappa(s,\ t) = \{\sin(\pi s)\exp(-\pi^2 t) + 1 - s\}^2 \tag{4.18}$$

Fig. 2 shows the curvature distributions of the deformed curves the shapes of the curves deformed by log-aesthetic flow.

5. Discrete log-aesthetic flow

In this section we discuss smoothing of discretely defined free-form curves, or polylines. Based on the log-aesthetic

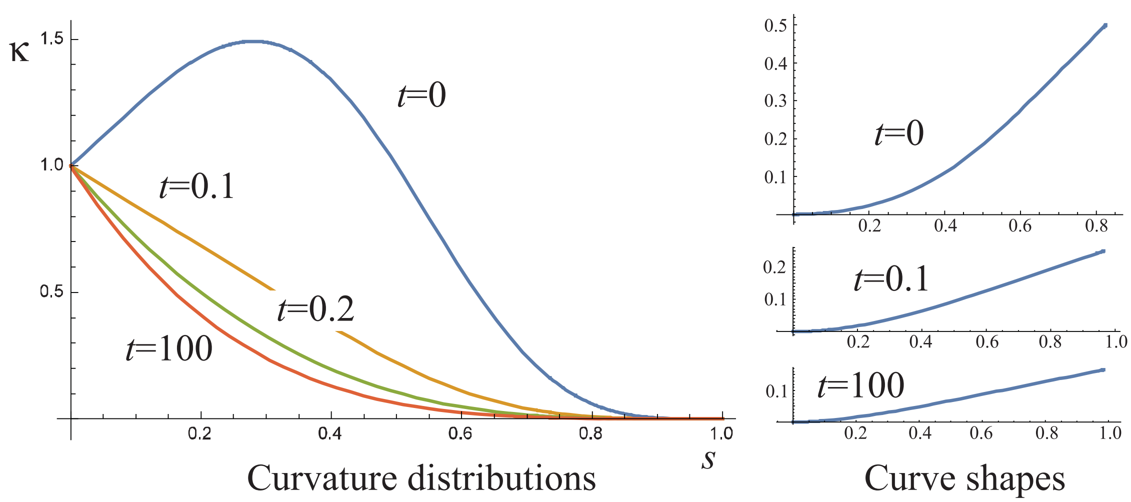


Figure 2. Curvature distributions and their curve shapes deformed by log-aesthetic flow.

flow based on the energy, we update the positions of the vertices of a polyline by the discretized partial differential equation derived from Eqn. (4.1). In this section we deal with the closed curve case as Crane et al. [1]. For a given closed sequence of points, the processes are summarized as follows:

Step 1. Evaluate curvature discretely using three consecutive points. The discrete curvature can be evaluated by

$$\kappa_i = \frac{2\theta_i}{l_{i-1,i} + l_{i,i+1}}. \tag{5.1}$$

where κ_i is a pointwise curvature at point f_i, θ_i is an angle between two consecutive edges, and $l_{i,i+1}$ is a length between f_i and f_{i+1} as shown in 3(a).

Step 2. Calculate a time derivative by $\dot{\kappa}_i = -2\kappa_i$.

Step 3. Build a constraint basis $\{\widehat{c}_i\}$ via the Gram-Schmidt orthonormalization.

Step 4. Project flow onto constraints $\dot{\kappa}_i \leftarrow \dot{\kappa}_i - \sum_i \ll \dot{\kappa}_i, \hat{c}_i \gg \hat{c}_i$ where double brackets $\ll \cdot, \cdot \gg$ denotes the inner product.

Step 5. Take an explicit Euler step as $\kappa_i \leftarrow \kappa_i + \tau\kappa_i$.

Step 6. Recover tangents by integrating κ_i.

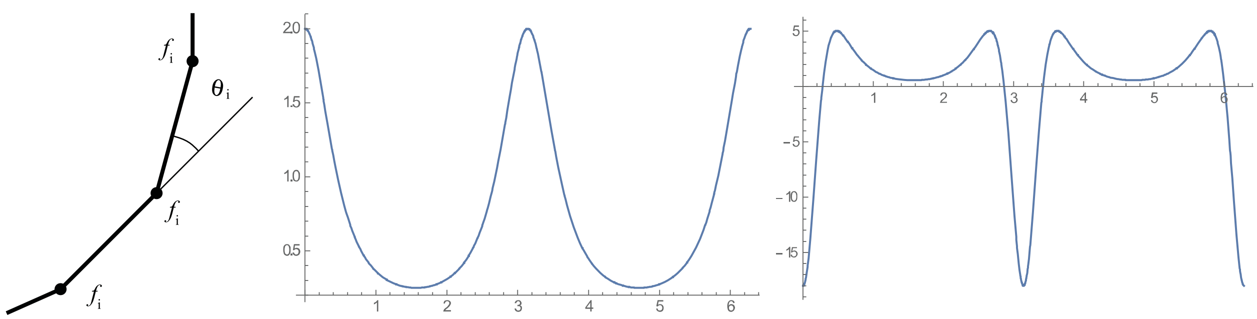
Step 7. Recover positions by solving a Laplacian equation $\Delta f = \nabla \cdot T$

Fig. 3(b). and (c). show the curvature and second derivative of curvature with respect to the arc length of an ellipse $(x/2)^2 + y^2 = 1$, respectively. The curvature of the ellipse expressed by $(a \cos\theta, b\sin\theta)$ is given by

$$\kappa = \frac{ab}{(a^2\sin^2\theta + b^2\cos^2\theta)^{\frac{3}{2}}} \tag{5.2}$$

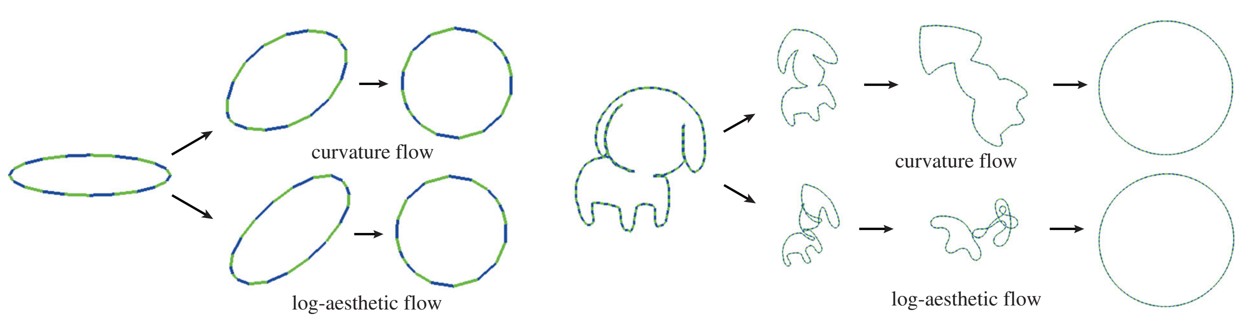
They indicate that log-aesthetic flow will reduce maximum curvature strongly and also try to increase curvature of the parts next to those of the maximum curvature. Fig. 4(a). shows the deformation processes of an ellipse by curvature and log-aesthetic flows. The curvature flow rapidly deforms the ellipse to a circle, but the log-aesthetic flow tries to keep its shape by increasing curvature of the parts next to those of the maximum curvature although it reduces the maximum curvature. Although the curve converges more quickly by curvature flow than by log-aesthetic flow, their final shapes are the same: a circle.

Fig. 4(b). shows a comparison between curvature flow and log-aesthetic flow applied to a more complicated shape. The final shapes are also circles in this case. If we interpret that the curve deformation is induced by log-aesthetic flow by heat conduction, the result is very natural because if we have a circular metal rod and keep



(a) Sequence of points (b) Curvature (c) Second derivative of curvature

Figure 3. Point sequence and curvature: (a) point sequence, (b) curvature of an ellipse, (c) second derivative of curvature shown in (a).



(a) Ellipse deformation (b) A dog example

Figure 4. Smoothing shapes by curvature flow and log-aesthetic flow.

one point of the rod at some temperature, the heat is conducted all over the rod and the temperature will be the same everywhere.

6. Conclusions and future work

In this research, we have proposed the concept of log-aesthetic flow to make free-form curves “log-aesthetic” based on curve shortening flow and curvature flow. We have proposed new smoothing methods that can handle analytically defined continuous curves as well as discrete polylines. Smoothing by curvature flow is very popular among CG and CAD communities and log-aesthetic flow will be another choice for smoothing based on a physical law different from that of curvature flow. We have one degree of freedom α to control smoothing for log-aesthetic flow and can expect the completely smoothed shape of a given curve, which means the shape obtained by fairing, to be log-aesthetic curve. Since log-aesthetic flow is basically governed by the heat conduction equation, which has been well studied in both physics and mechanical engineering, its effects are easily inferred by the designers and we hope that it will be useful for practical aesthetic design. For future work, we would like to extend our method based on log-aesthetic flow for space curves and free-form surfaces.

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ORCID

Kenjiro T. Miura http://orcid.org/0000-0001-9326-3130
Sho Suzuki http://orcid.org/0000-0002-2607-7204
R.U. Gobithaasan http://orcid.org/0000-0003-3077-8772
Péter Salvi http://orcid.org/0000-0003-2456-2051
Shin Usuki http://orcid.org/0000-0003-1165-5507

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