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Finite Element Formulation for Semi-Rigid Beam-To-Column Connections Using Potential Energy Approach

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Abstract

This paper presents the behaviour of semi-rigid connections in steel frame analysis by utilizing the total potential energy principal. A finite element model is proposed to consider the behaviour of the semi-rigid connection. The proposed finite element method is compared with the existing method proposed by other researchers. Both linear and non-linear analyses are performed to acquire the solution of steel frame under elastic condition. The results of comparison agreed well with other method proposed by other researchers. The results of the proposed finite element method is also compared with the available experimental results and found that the linear analysis agrees well with the experimental results. Therefore, it is concluded that the behaviour of the semi-rigid connection is steel framing can be predicted using potential energy approach.

Keywords: Finite element method; semi-rigid connections; potential energy; connection stiffness

1. Introduction

Traditionally, steel frame is designed by assuming that beam-to-column joints are rigid or pinned. Rigid joints are described where no relative rotation occurs between the connected members while pinned joints imply that no moment is transmitted from beam-to- column. However, semi-rigid joint requires both relative rotation and moment in the analysis of steel frame.

In the past two decades, considerable researches have been carried out to access the actual behaviour of steel frame connections. Li et. al. [1] used a Connection Element Method (CEM) for analysis of semi-rigid frames. A general procedure was presented incorporating the effects of joint flexibility into standards method. The method allowed for joint flexibility associated with the degree of freedoms and considered coupling between deformations. Kishi et. al. [2] provided an analytical evaluation of the Eurocode 3 classification on the three types of beam-to-column connections in steel construction. It was found that most connections developed semi rigid behaviour for drift check at service load, but is feasible for strength check at factored load. Therefore, the same connection may be considered semi-rigid at service load level but flexible at factored load. Based on this consideration, two levels of definition have been established. Bahaari and Sherbourne [3] used finite element method to predict the behaviour of end-plate bolted connection. The study presents the characteristics of the model together with a multiple-regression analysis procedure for deriving the parameters in terms of connection's description. It was found that the Richard-Abbot power function has proved appropriate for expressing M-θ relationship. The function describes the behaviour of the connection in tension region of column flange and developed standardized function.

Morfidis and Avramidis [4] developed a formulation of a generalized beam element on a two-parameter elastic foundation with semi-rigid connections. The derivation of element of stiffness matrix was based on the exact solution of the differential equation governing displacements. The stiffness matrix was developed in two stages with the exact stiffness matrix of median segment formed in first stage. In second stage, relations between the coefficients of the stiffness matrix of the median segment to the coefficients of the stiffness matrix of the new element were formulated. The study indicated that versatility in the analysis of any type of linear structures and simplicity offers the possibility of modelling the foundation beams and steel structures with semi-rigid connections. Hadianfard and Razani [5] used Monte Carlo simulation technique to illustrate the importance on the effect of semi-rigid connections behaviour in reliability of steel frames. The study was also considered the effect of semi-rigid connections behaviour in finite element analysis. The calculation of the probability of failure was made by modelling the connections behaviour in reliability analysis of steel frames. It was found that there were substantial differences in the result or reliability analysis between the more realistic semi-rigid connections and the cases in which extreme assumptions of fully rigid or fully pinned connection were used.

Al-Jabri [6] proposed a component-based model to predict the behaviour of flexible end-plate connections with increasing temperatures. Connection elements were treated as springs with pre-defined stiffness and strength and modelled by assembling the contributions of individual components. In general the component model is capable of predicting the connection response at both ambient and elevated temperatures to a reasonable accuracy especially in the elastic zone. Cabrero and Bayo [7] introduced a practical design method for semi-rigid joints. The method allowed optimizing not only the size of the structured profiles, but also the joint design. The method also provides checks on feasibility and suitability of a connection design. Cost estimations for the design have been also carried out and proved as the most cost effective solution in comparison to traditional types of joints. By means of this method, effectiveness of semi-rigid construction can be in cooperated in more competitive and efficient manner. The method was also used to obtain a pre-design joint adequate to the required stiffness and resistance. Law et. al. [8] investigated the dynamic behaviour of a bolted joint which has flexibility in both tangential and rotational directions. The study presented the formulation of the hybrid beam-column element including the end springs and dynamic behaviour of cantilever beam with non-linear semi-rigid joint. A hybrid beam-column element including the virtual connection end spring element representing the joint was also presented. It was found that the natural frequencies and mode shapes could be maintained when both shear and moment stiffness's were greater than 10^8 N/m or 10^8 Nm/rad. It was also found the joint location have no significant effect on natural frequency provided that shear and moment stiffness are above 10^8 N/m or 10^8 Nm/rad respectively. Castro et. al. [9] proposed a new approach for modelling steel and composite joints within frame analysis. The study considered the effect of different boundary conditions, as well as shear and flexural deformation modes, in evaluating the elastic and inelastic response.

The approach was validated by comparing with available experiment results in addition to more details continuum finite element analysis. The study describes the implementation of the suggested approach within frame analysis procedure, and substantiates the important role played by panel zone in response to moment frames under lateral loading conditions. Bayo et. al. [10] proposed a new component-based approach to model internal and external semi rigid connections for global analysis of steel and composite frames. The method was based on a finite dimension of elastic-plastic four - nodes joint element. Considerations that take into account were the deformation characteristic (components in Euro code) and internal forces coming from the beams and columns occurred at joints. In this study, a joint elastic-plastic element has been proposed for global structural analyses. This element takes into consideration all the deformation components such as panel zone, internal forces and join eccentricities.

Vellasco et. al. [11] used ANSYS software to model the semi-rigid connection by considering parameters such as connection stiffness and strength, structural system (steel or composite) and lateral frame stability. Results obtained were summarized to access the economic potential and efficiency of semi-rigid connections. Recently, a work done by Ihadoudene et. al. [12] on semi rigid effect or behaviour takes into account extra elements between beam and column connections. Related to this statement, Ihaddoune et. al. have proposed mechanical model based on analogy of three springs (two translational and one rotational) attached to the end of bar elements. The non-deformable nodes were used to describe relative displacements and rotations between the nodes and elements. Therefore, by taking into consideration the flexibility of the connection in steel frames, a simple method of analysis and design was provided through a mechanical model for the joints. It offers a simple, direct and versatile approach to the structural analysis with semi-rigid joints as compared to the complex models and cumbersome non-linear procedure in use.

2. Finite element formulation on connection

The finite element formulation on connection will focus on the modelling of the connection by incorporation semi-rigid effect. A linear formulation of semi-rigid connection will be adopted to establish the modelling of the connection.

2.1 Connection modelling by incorporating semi-rigid effect

In semi-continuous and continuous constructions the connections should be able to transfer vertical shear and moment. When load is applied to the frame structure, it will develop moment to a beam-to-column connection, the connected beam and column rotates relative to each other by an amount of rotation designated as theta "θ". There are several models to idealize the relationship between moment and rotation with moment-rotation diagram. For example, linear models, polynomial models, power models, exponential models, are among popular models that have been developed to describe moment-rotation relationship. However, in this paper potential energy approach incorporating with spring mechanism is adopted to model the semi-rigid connections.

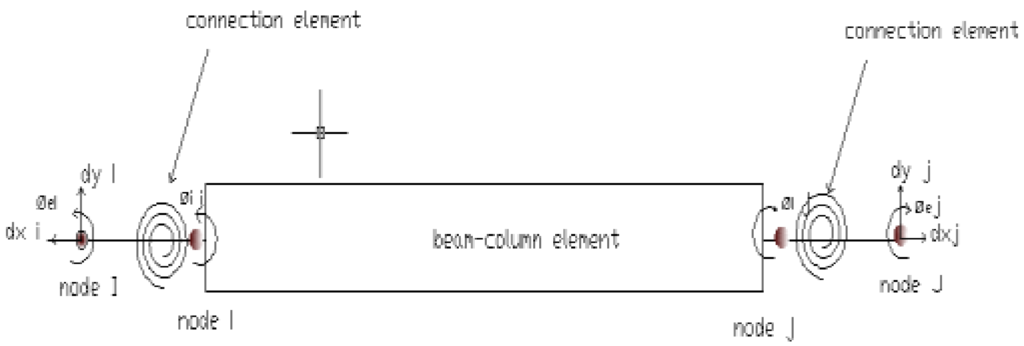

Fig.1: Beam-to-column element

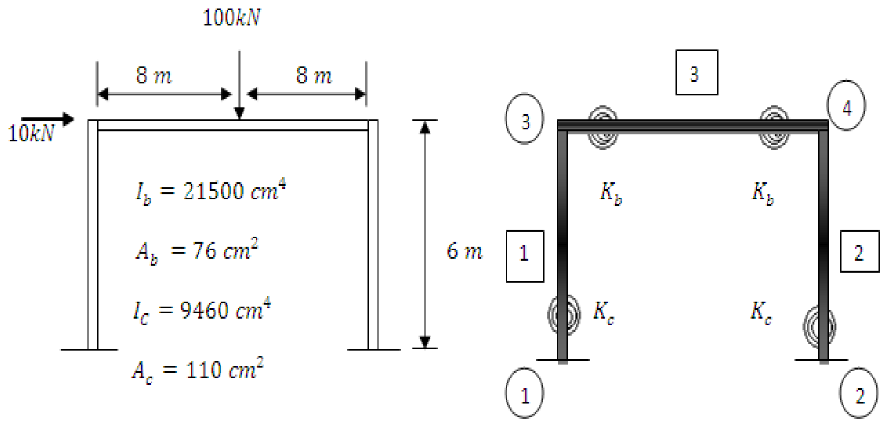

Fig. 2: One bay one storey frame with end connections spring elements

The proposed semi-rigid joint is shown in Fig. 1. This element incorporating both in-plane shear and moment flexibilities modelled as a virtual connection spring element attached to the end of a beam-to-column element to form a hybrid element as shown in Fig. 1. This connection of spring element is located at member ends and is assumed infinitely small. Nodes I and J are external nodes and nodes i and j are internal nodes respectively as shown in Fig 1. The external nodes are idealized by connecting them to the outer node or global reference system while the internal nodes are located at the beam-to-column element. The actual rotation of the system is simply defined by excessive rotation between external node and internal node.

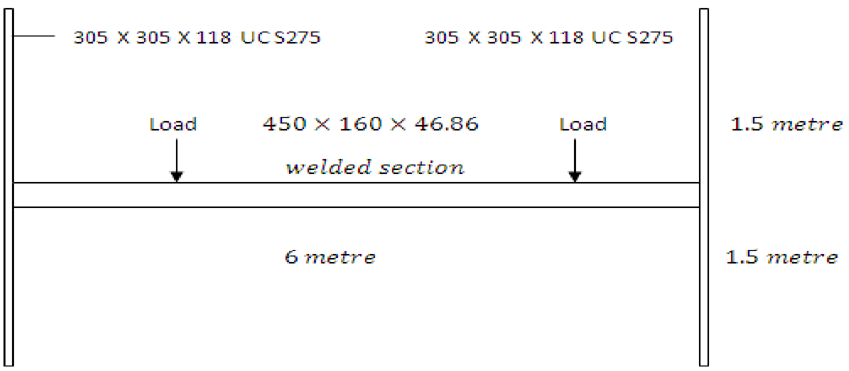

Fig.3: Overview of sub-assemblage test frame

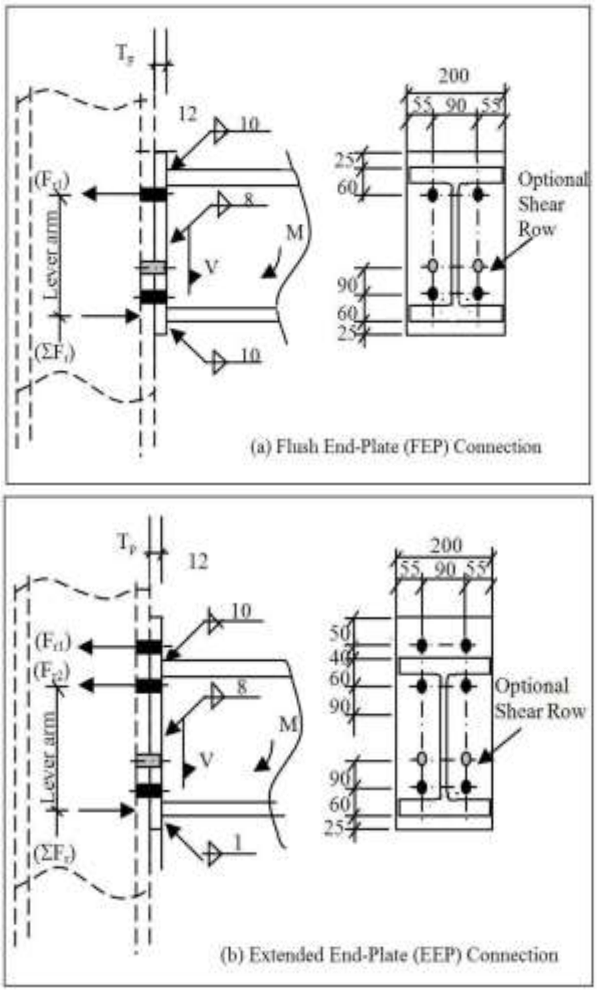

Fig. 4: Flush and Extended End-Plate connections as semi-rigid connections

2.2 Linear formulation of semi-rigid connections

The approach to establish the linear formulation of semi-rigid connections is by means of stiffness matrix. The stiffness matrix is derived in two stages. For the first stage of derivation, the stiffness matrix of beam-to-column element is formed.

This stiffness matrix is adopted from Bernoulli or Timoshenko beam element available from many sources. For the second stage, which is the main objective of this paper, the stationery potential energy principle is used to formulate the connection element stiffness matrix and corresponding equilibrium equation. The total potential energy functional, $\prod$ is defined as follows:

$$\Pi = U + V$$

where U is the system strain energy and V is the load total potential. The system strain energy, U, can be expressed in terms of spring stiffness as spring strain energy and its relative displacement is defined as follows:

$$Spring\ strain\ energy, U = {}^1\!/_2\, k\, \Delta\, \theta^2$$

The term θ can be idealized as θ_e *and* θ_i to represent external and internal rotations at the nodes respectively. Therefore, system strain energy can be expressed as

$$U = {}^1\!/_2\, k\, (\theta_e - \theta_i)^2 \quad \text{or} \quad U = {}^1\!/_2\, k\, \left(\theta_e - 2\theta_e\theta_i + \theta_i{}^2\right) \tag{1}$$

Differentiate spring strain energy, U with respected to θ_e,

$$\frac{\delta U}{\delta \theta_e} = {}^1\!/_2\, k\, (2\theta_e - 2\theta_i) \tag{2}$$

Once again, differentiate spring strain energy, U with respected to θ_i

$$\frac{\delta U}{\delta \theta_i} = {}^1\!/_2\, k\, (-2\theta_e + 2\theta_i) \tag{3}$$

Rewriting Eq. 2 and Eq. 3 above in matrix form, the stiffness matrix of a connection spring can be written as:

$$\begin{bmatrix} S & -S \\ -S & S \end{bmatrix} \begin{bmatrix} \theta_e \\ \theta_i \end{bmatrix} = \begin{bmatrix} M_e \\ M_i \end{bmatrix} \tag{4}$$

2.3 Stiffness matrix

Stiffness matrix for a complete structure in solving displacement is first developed, transformed and assembled. In order to develop complete stiffness matrix including semi-rigid effects, the stiffness connection spring, k_c is included in the system. This leads to the adoption of connection spring elements as in Eqn. 5.

$$K_c = \begin{bmatrix} \frac{4EI}{L} + S_1 & \frac{2EI}{L} & -S_1 & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} + S_2 & 0 & -S_2 \\ -S_1 & 0 & S_2 & 0 \\ 0 & -S_2 & 0 & S_2 \end{bmatrix} \tag{5}$$

Therefore, a hybrid element can be obtained by directly adding the connections stiffness, K_c to the element of bending stiffness in Eqn. 6 as:

$$K_e = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & 0 & 0 \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L}+S_1 & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & -S_1 & 0 \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{4EI}{L}+S_2 & 0 & -\frac{6EI}{L^2} & 0 & -S_2 \\ -\frac{EA}{L} & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & 0 & 0 \\ 0 & 0 & -S_1 & 0 & 0 & 0 & S_1 & 0 \\ 0 & 0 & 0 & -S_2 & 0 & 0 & 0 & S_2 \end{bmatrix} \quad (6)$$

Kc

In global reference, the stiffness matrix is obtained as: $K = T_e^T . K_e . T_e$ (7)
in which T_e is the transformation stiffness matrix given in Eqn. 8 as:

$$T_e = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

3 Numerical Examples

In order to validate the proposed model, comparison to previous literature is made. This validation is done by solving problem for examples given in Case 1: Linear analysis for simple portal frame, Case 2: Linear analysis of sub-assemblage frame, and Case 3: Non-Linear analysis of sub-assemblage frame.

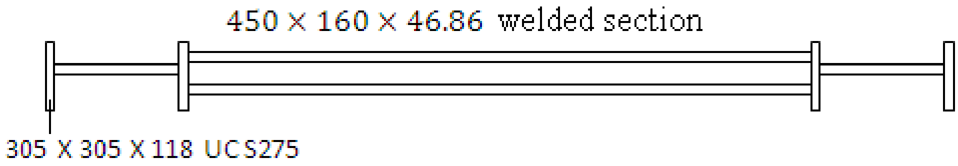

Fig.5: Detailed out cross-section (plan view) of conventional column for N9and N13

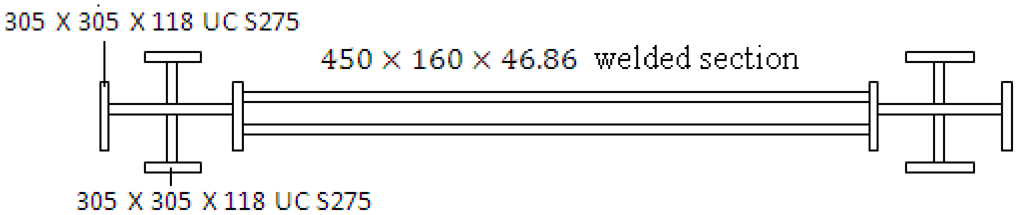

Fig.6: Detailed out cross-section (plan view) of cruciform column for N10 and N14

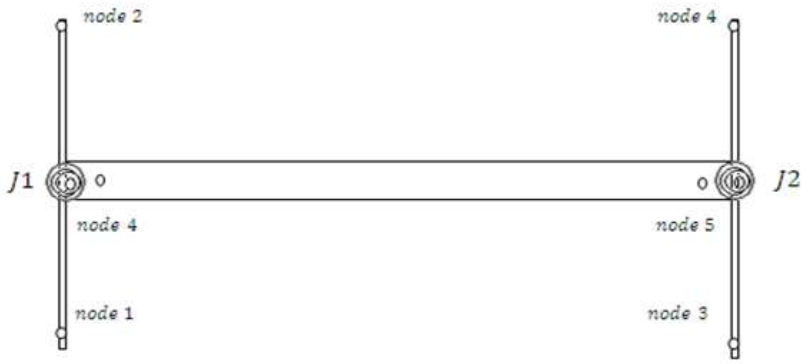

Fig 7: Locations of the connections J1, J2 and node numbering

3.1 Case 1: Linear analysis of simple portal frame

For numerical analysis in Case 1, a one bay one storey frame is chosen (see Fig.2). This simple frame has been proposed as a benchmark to check the behaviour of semi-rigid connection as well as the finite element formulation itself. Therefore, a frame of width 16m and of height 6m [12, 13] subjected to vertical point load of 100 kN at mid-span of the beam and horizontal point load of 10kN at top left column is analysed with different value of connection's stiffness. The connection stiffness's used in this example; K_b and K_c are the connection stiffness's at the beam-to-column respectively for semi-rigid connection. However, for rigid connection, the connection stiffness is idealized as infinity. In pinned connection case, the connection stiffness is taken as zero.

The characteristic of the connection stiffness is depicted in Table 1. For demonstrative purpose, beam stiffness is taken as the stiffness of semi rigid connection.

Table 1: Case and type of connections

Case	Type of connection	*Kb*	*Kc*
A	Rigid	∞	∞
B	Semi-rigid	$4EI_b/L_b$	∞
C	Semi-rigid	$4EI_b/L_b$	EI_c/L_c

A research done by Chan SL [13] simply used spring element at the joint to model the connection behaviour. The connection at the node should be in equilibrium and satisfy the stiffness requirement for rotation at two ends. However, a research work carried out by Ihaddoune et al [12], a mechanical model is used to represent semi-rigid behaviour at connection node. The flexibility method is used in the analysis to solve for the reaction at the joint. It can be observed in Table 2 that the bending moments at node 1 to node 4 obtained from the proposed formulation is very similar to Chan SL [13] and Ihaddoune et al [12]. The results show that the proposed connection element is in good agreement with other research works [12, 13].

Table 2: Absolute maximum moments

Type of connection	Properties		Result of analysis (Moments, kNm)		
			Current Practice	Chan.SL [14]	Ihaddoune [13]
Rigid connection	$Kb = \infty$ $Kc = \infty$	M_{13}	52.23	52.20	52.20
		M_{31}	127.50	127.60	127.60
		M_{24}	87.14	87.13	87.10
		M_{42}	152.58	152.70	152.70
Semi-rigid beam-to-column connection	$Kb = 4EI_b/L_b$ $Kc = \infty$	M_{13}	31.68	31.70	31.90
		M_{31}	93.65	93.60	93.70
		M_{24}	71.53	71.50	71.80
		M_{42}	113.79	113.80	113.90
Semi-rigid beam-to-column connection and column based	$Kb=4EI_b/L_b$ $Kc = EI_c/L_c$	M_{13}	0.32	0.30	0.34
		M_{31}	80.25	80.30	80.27
		M_{24}	24.16	24.20	24.18
		M_{42}	116.41	116.40	116.43

3.2 Case 2: Linear analysis of sub-assemblage frame

Two particular types of beam-to-column connection are used in the sub-assemblage study, i.e. flush end-plate (FEP) and extended end-plate (EEP) connections as shown in Fig. 4. For linear analysis, a series of four sub-assemblage tests is modelled based on their initial stiffness, $S_{j,\mathrm{ini}}$. The results obtained are compared with the experimental test carried out by Shek [14]. All sub-assemblages setting in this study comprised of 6m long of size 450 x 160 x 46.86 built-up beam section connected to two column sections and tested in H-frame manner. For these sub-assemblage tests, the type of beam-to-column connections used are extended end-plate (EEP) connection for specimens N9 and N10 and flush end-plate (FEP) connection for specimens N13 and N14 respectively. There are two tests designated as specimens N9 and N13 with built-up beam and hot-rolled column (UC) (see Fig. 5) and two tests designated as specimens N10 and N14 with built-up beam and built-up cruciform column (see Fig. 6). The results of the linear and non-linear analyses proposed are compared to experimental test results [14]. For linear analysis, only linear part of moment-rotation analysis is considered by using knee joint method. For this case, it is solved by considering the same scenarios as in Case 1. The characteristic of the connections are depicted in Table 3 and the locations of the connections are illustrated in Fig 7. The bending moment and mid-span deflection results from simulation of the beam and the comparison to experimental test to the proposed model are shown in Table 4. The locations of the nodes are referred to Fig 8.

Table 3. Characteristic of the connections

Specimen	Left connection initial stiffness (kNm/Rad), J1	Right connection initial stiffness (kNm/Rad), J2
N9	51563	36538
N10	77778	75000
N13	60000	52857
N14	93000	76470

In this case, the connection's stiffness is defined as initial stiffness, $S_{j,ini}$. For the linear case of sub-assemblage frame tests, there are two types of column that are used which are conventional column and cruciform column. Specimens N9 and N13 used conventional column (UC) while N10 and N14 used built-up cruciform column. The results of deflections show that finite element formulation is significantly accurate with conventional column (see Fig. 8); however it is less accurate when compared to built-up cruciform column (see Fig. 19).

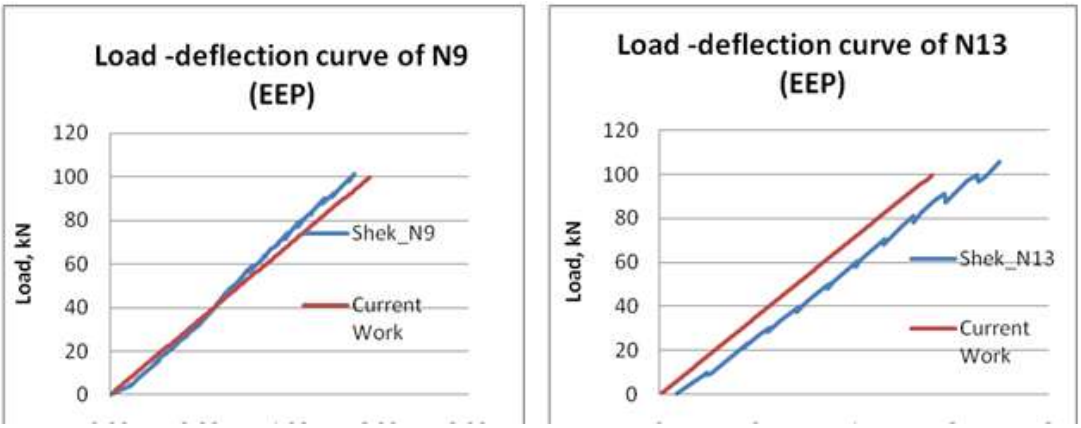

Fig. 8: Load-deflection curves between the proposed model and N9 and N13 specimen

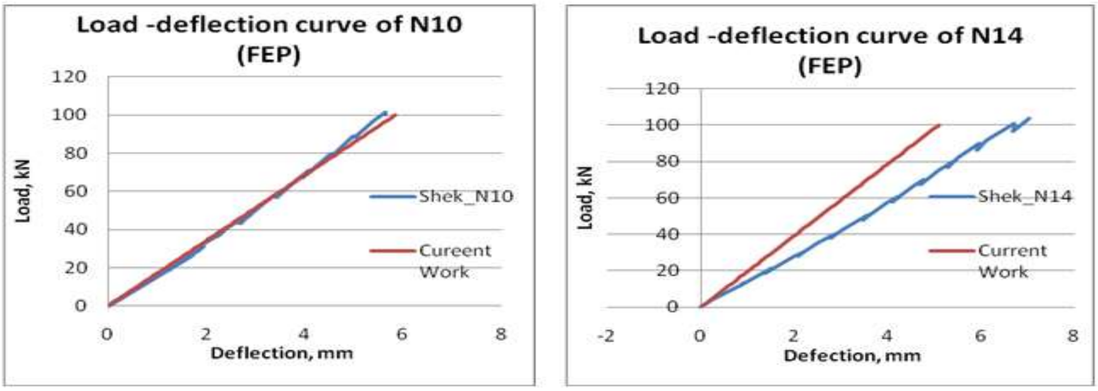

Fig. 9: Load-deflection curves between the proposed model and N10 and N14 specimen

On the other hand, Fig. 10 illustrates the comparison of moment rotation between the proposed model and the specimens N9 and N13 models (EEP) subjected to vertical force. Fig. 11 presents comparisons of moment-rotation curve between the proposed model and the tested specimens, N10 and N14. The overall comparisons show good correlation between the experimental test results and the proposed model as shown in Table 4.

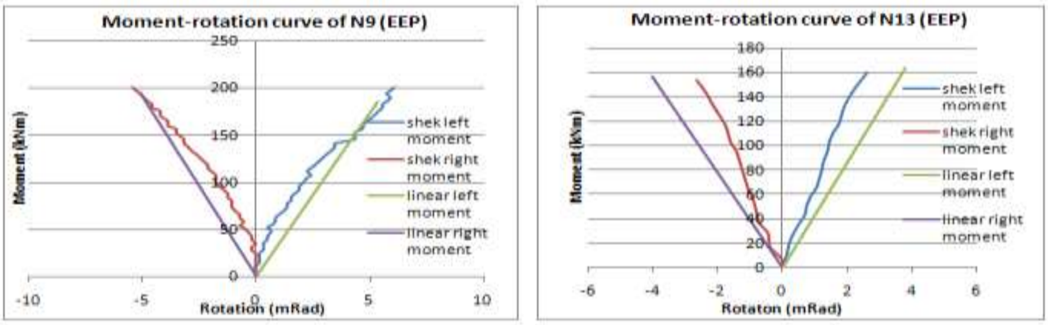

Fig. 10: Moment-rotation curves between the proposed model and N9 and N13 specimen

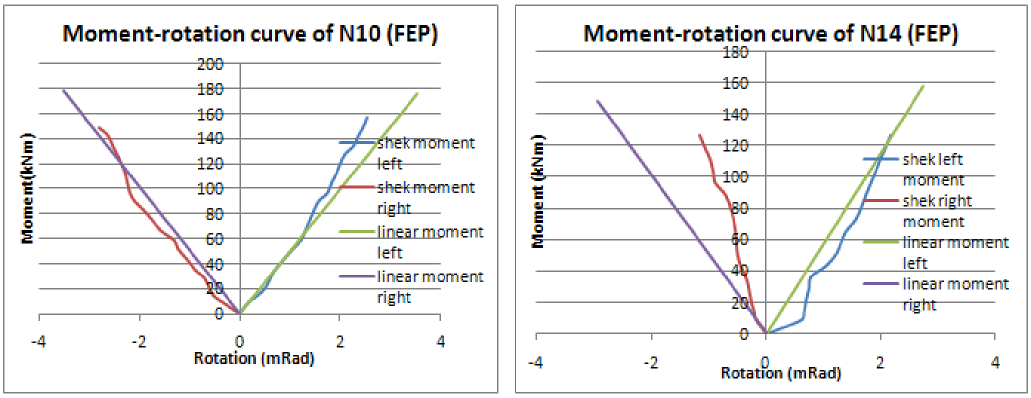

Fig. 11: Moment-rotation curves between the proposed model and N10 and N14 specimen

Table 4: Comparison of moments and deflections of the beam for case 2

Test number	Location	Proposed element moment (kNm)	Experimental moment (kNm)	Proposed element mid-span deflection (mm)	Experimental mid-span deflection (mm)
N9	J1	184.57	165.00	5.80	5.46
	J2	195.14	190.00		
N10	J1	176.09	140.00	5.84	5.61

Table 4 (continued): Comparison of moments and deflections of the beam for case 2

	J2	178.04	165.00		
N13	J1	163.88	128.00	5.61	6.60
	J2	156.36	149.00		
N14	J1	157.04	125.00	5.09	7.40
	J2	148.59	130.00		

3.3 Nonlinear Analysis

Flexibility of beam-to-column connection is characterized by a moment-rotation curve as non-linear over entire imposed load. The relationship is actually non-linear for all types of connections and varies depending on connection flexibility. Fig. 12 shows various proposed models to fit a moment rotation curve.

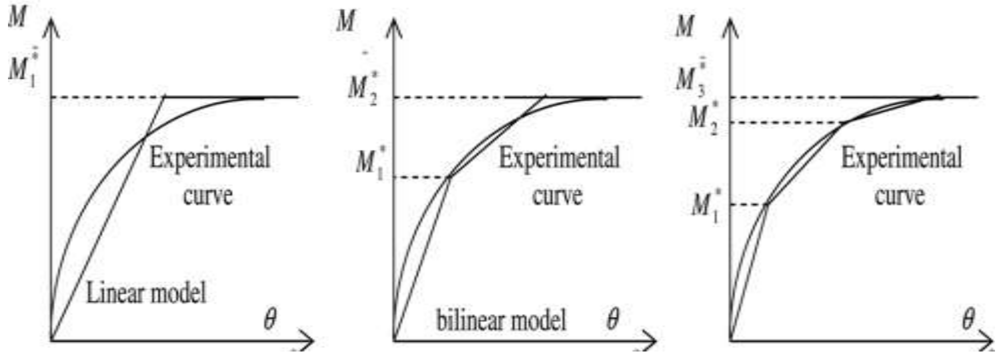

Fig. 12: Various models proposed for moment rotation curve

Under a monotonous loading, the non-linear relation between moment and rotation can be expressed as in Eqn. 9:

$$M = k\theta \tag{9}$$

The relation can be expressed in each stage as:

$$M^{(i+1)} = M_{ini} + K^{(i+1)}\theta \tag{10}$$

where $M^{(i+1)}$ is the moment limitation at ith stage:

For the first stage: $M^{(1)} = K^{(1)}\theta\,,\quad M_{ini} = 0$ (11)

For the second stage: $M^{(2)} = M_{01} + K^{(2)}\theta$ (12)

In which M_{01}= *y-axis* intersection

3.4 Steps to include tri-linear stiffness

This process is divided into some steps and it is based on shape of the moment-rotation curve. For the first stage, all joints having the same stiffness $K^{(1)}$. Then, the load increases gradually until reach permitted moment M_1* as indicated in Fig. 12. In the second stage, the joint has stiffness $K^{(2)}$. The loads continue to increase up to a level where the moment reaches M_2*. A step by step process is continued until the sum of load increment reached M_3*.

3.4 Case 3: Non-linear analysis of sub-assemblage frame

The same frame as in Case 2 is analysed using tri-linear approach and the results are compared with experimental test results carried out by Shek [14]. In this case, three stages of linear stiffness (see Fig. 12) are used in order to solve sub-assemblage frame as depicted in Fig 13 and Fig.14. The bending moments and deflections, resulting from the simulations at the joints and middle of the beams are shown in Fig. 13 and Fig. 14. The location of the joints is referred to Fig. 7. Results from the analysis show similar pattern to that of Case 2. The difference to the bending moments and deflections is due to the use of three stages of linear stiffness instead of only one stages of linear stiffness. It is quite significant for both the beams and the joints. The results of load versus deflection show significant difference between the model used conventional column (N9) (see Fig. 13) and the model used cruciform column (N14) (see Fig. 14), where the result of conventional column is more accurate than that of cruciform column.

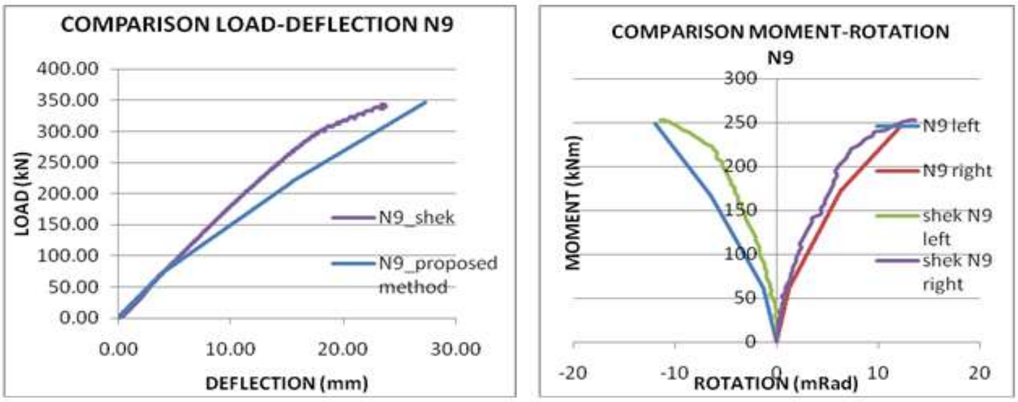

Fig. 13: Comparison of load-deflection and moment-rotation curve between the proposed model and N9 specimen

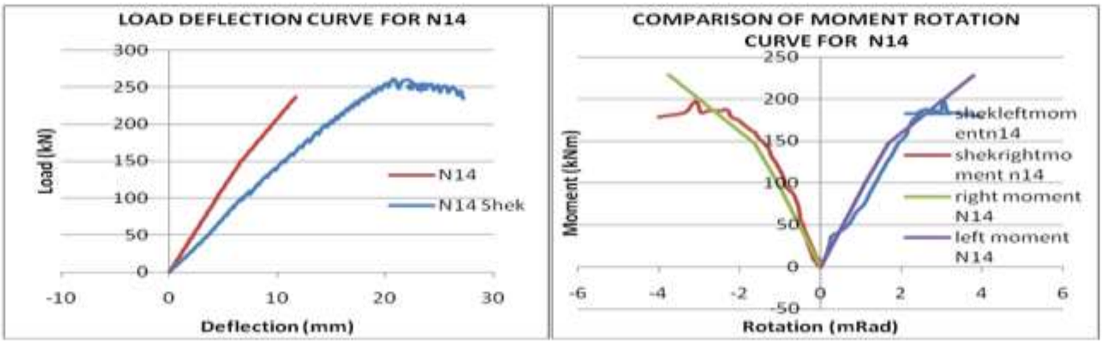

Fig. 14: Load-deflection and moment-rotation curves between the proposed model and N14

3.5 Comparison between Case 2 and Case 3 for N9 (conventional column)

In this section, the efficiency and usefulness of the stiffness effect are presented. In order to compare the result based on the difference between linear analysis, tri-linear analysis and experimental tests results, the same sub-assemblage frame examples have been adopted. Table 5 and Table 6 summaries the comparison of results between linear analysis, tri-linear analysis and experimental tests results [14].

Table 5: Moment and rotation for experimental, linear and tri-linear model for N9 specimen

Moment (kNm)	Rotation (radian)				
	Experimental result	Case 2 (linear)		Case 3 (tri-linear)	
		result	difference	result	difference
Left moment	9.25	4.98	4.27	11.41	2.16
Right moment	9.60	4.43	5.17	11.72	2.12

Table 6: Load-deflection for experimental, linear and tri-linear model for N9 specimen

Load (kN)	Deflection (mm)				
	Experimental result	Case 2 (linear)		Case 3 (tri-linear)	
		result	difference	result	difference
340.00	23.77	18.04	5.73	26.70	2.93

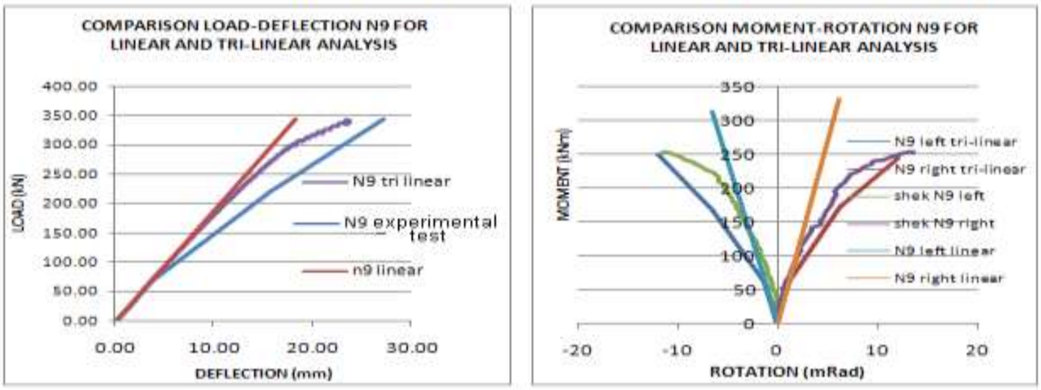

Fig. 15: Load-deflection and moment-rotation curves between the experimental tests, linear model and tri-linear model for N9 specimen

From the results of analysis, it becomes obvious that in the tri-linear analysis there is rather significant effect to moments and deflections if compared to the linear analysis as shown in Fig. 15. A simple inspection of Fig. 15 indicates that by using three types of stiffness, the result obtained is more closed to the actual experimental test as compared to the linear analysis result. Therefore, by using this tri-linear approach we can avoid over estimation of the results and make the analysis more effective and accurate. This leads to the conclusion that the semi-rigid model using the proposed potential energy approach for the tri-linear analysis can be adequately modelled as sub-assemblage frame with semi-rigid connections.

4. Conclusions

A new formulation of semi-rigid beam to column connections for use of steel frame analysis has been presented. The element is comprised of a beam-to-column element and connection element. The connection element between flexible beam and column is chosen to be either rigid, semi-rigid or pinned. The versatility of the present element render its suitability for implementation in structural analysis computer programs where the ability to choose the appropriate condition of connections.

The usefulness of this new formulation has been illustrated by three examples designated as Case 1: Linear analysis of simple portal frame, Case 2: Linear analysis of sub-assemblage frame, and Case 3: Non-linear analysis of sub-assemblage frame. In the Case 1, the effectiveness of the new element in modelling steel portal frame is shown with comparison again Chan SL [13] and Ihaddoune et al [12]. The results show that the proposed connection element is in good agreement with other research works conducted by both Chan SL [13] and Ihaddoune et al [12]. In Case 2, the reliability of the new element is compared with the experimental test results carried out by Shek [14]. The comparison results show that it is more accurate to use conventional column element as compared to cruciform column element which means that an extra term should be provided to remodel cruciform column. In Case 3, the sub-assemblage frame analysis is done by divided the stage of analysis into three stages. The results obtained show that accuracy of result from Case 3 is more accurate when compared to the second case. The case examples presented indicate that the versatility of the new formulation in the analysis of structures, the simplicity which it offers in the modelling of steel structure with semi-rigid connection. This leads to the conclusion that the semi-rigid model using the proposed potential energy approach for the tri-linear analysis can be accurately modelled sub-assemblage frame with semi-rigid connections.

References

[1] T. Q. Li, B. S. Choo, D. A. Nethercot, Connection element method for the analysis of semi-rigid frame, *J. Construct. Steel Resesarch*, **32** (1995), 143-171. http://dx.doi.org/10.1016/0143-974x(95)93170-9

[2] N. Kishi, R. Hassan, W. F. Chen, Y. Goto, Study of Eurocode 3 steel connection classification, *Engineering Structures*, **19** (1997), no. 9, 772-779. http://dx.doi.org/10.1016/s0141-0296(96)00151-4

[3] M. R. Bahaari, A. N. Sherbourne, Finite element prediction of end plate bolted connection behaviour. II: analytic formulation, *Journal of Structural Engineering*, **123** (1997), no, 2, 165-175. http://dx.doi.org/10.1061/(asce)0733-9445(1997)123:2(165)

[4] K. Morfidis, I. E. Avramidis, Formulation of a generalized beam element on a two-parameter elastic foundation with semi-rigid connections and rigid offsets, *Computers and Structures*, **80** (2002), 1919-1934. http://dx.doi.org/10.1016/s0045-7949(02)00226-2

[5] M. A. Hadianfard, and R. Razani, Effects of semi-rigid behavior of connections in the reliability of steel frames, *Structural Safety*, **25** (2003), no. 2, 123-138. http://dx.doi.org/10.1016/s0167-4730(02)00046-2

[6] K.S. Al-Jabri, Component-based model of the behaviour of flexible end-plate connections at elevated temperatures, *Composite Structures*, **66** (2004), no. 1-4, 215-221. http://dx.doi.org/10.1016/j.compstruct.2004.04.040

[7] J.M. Cabrero, and E. Bayo, Development of practical design methods for steel structures with semi-rigid connections, *Engineering Structures*, **27** (2005), no. 8, 1125-1137.

http://dx.doi.org/10.1016/j.engstruct.2005.02.017

[8] S. S. Law, Z. M. Wu, S. L. Chan, Transverse natural vibration of a beam with an internal joint carrying in-plane flexibilities, *Journal of Engineering Mechanics*, **131** (2005), 80-87. http://dx.doi.org/10.1061/(asce)0733-9399(2005)131:1(80)

[9] J.M. Castro, A.Y. Elghazouli, and B.A. Izzuddin, Modelling of the panel zone in steel and composite moment frames, *Engineering Structures*, **27** (2005), no. 1, 129-144. http://dx.doi.org/10.1016/j.engstruct.2004.09.008

[10] E. Bayo, J.M. Cabrero, and B. Gil, An effective component-based method to model semi-rigid connections for the global analysis of steel and composite structures, *Engineering Structures*, **28** (2006), no. 1, 97-108. http://dx.doi.org/10.1016/j.engstruct.2005.08.001

[11] P.C.G. Da S. Vellasco et al., A parametric analysis of steel and composite portal frames with semi-rigid connections, *Engineering Structures*, **28** (2006), no. 4, 543-556. http://dx.doi.org/10.1016/j.engstruct.2005.09.010

[12] A.N.T. Ihaddoudène, M. Saidani, and M. Chemrouk, Mechanical model for the analysis of steel frames with semi rigid joints, *Journal of Constructional Steel Research*, **65** (2009), no. 3, 631-640. http://dx.doi.org/10.1016/j.jcsr.2008.08.010

[14] P. N. Shek, *Structural Behaviour of Built-Up Steel Sections Using Partial Strength Connection*, Ph.D Thesis, Universiti Teknologi Malaysia, Skudai, Johor, Malaysia, 2009.

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