

## A NEW DIMENSION OF PROFIT-SHARING RATIO DETERMINATION OF AITAB CONTRACT MATHEMATICAL MODELLING USING STOCHASTIC OPTIMISATION APPROACH

NURFADHLINA ABDUL HALIM<sup>1\*</sup>, SAIFUL HAFIZAH JAAMAN<sup>2</sup>, NORISZURA ISMAIL<sup>3</sup> AND ROKIAH AHMAD<sup>4</sup>

<sup>1</sup>Department of Mathematics, Universiti Malaysia Terengganu, 21030 Kuala Terengganu, Terengganu, Malaysia. <sup>2,3,4</sup> Centre for Modeling and Data Analysis, School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia.

\*Corresponding author: lina@umt.edu.my

**Abstract:** This paper discusses the problem of determining the profit-sharing ratio for Islamic hire-purchase contract named *Al-Ijara Thumma Al-Bai'* or AITAB. AITAB originated from and is widely used in Malaysia. This paper presents an approach which maximises the profits of the participating parties in the hire-purchase instrument by implementing the fair profit-sharing ratio determination. First, the AITAB agreement is discussed followed by the mathematical formulation of the proposed profit-sharing ratio (PSR) determination using stochastic programming. The result of this study is the mathematical formulation of PSR for the AITAB contract which is more appropriate and *Shari'ah* compliant if compared to the formulation currently used.

**KEYWORDS:** Islamic finance, Islamic hire-purchase, profit-sharing ratio determination

### Introduction

The economic system in Malaysia is unique with the conventional system and the Islamic system operating alongside each other (Seif *et al.*, 2007). Compared to the conventional system which is well established, the Islamic system has only played a significant role in the banking and finance sector for the last 30 years (Muhammad, 2007). Even though the Islamic system can be considered as just starting out, it has played a significant role, with Islamic banking and finance total assets exceeding USD200 billion and growing at an annual rate of 6% a year (Amir and Ali, 2007). However, in order to compete with the conventional banks in attracting customers and investors, Islamic banks will need to toil harder. This has resulted in the creation of more attractive and innovative products. Even with these products, as well as a flourishing reception, there are still some improvements needed due to lack of *Shari'ah* (Islamic law) outline within the products (Irwani and Asyraf, 2006). According to Sudin (1996) the lack of *Shari'ah* outline is due to lack of experience, research and scarcity of literature. Furthermore, Islamic products are not

able to compete with the captive products and interest rate of the conventional banks.

Hire-purchase is an instrument offered by banks in Malaysia. The agreement includes letting of goods with an option for lessee to purchase the goods through instalments. Hire-Purchase Act 1967 (Act 212) state that hire-purchase agreements are valid to all consumer's goods and motor vehicles (Legal Research Board, 2009). Islamic hire-purchase agreement in Malaysia is known as *Al-Ijara Thumma Al-Bai'* or AITAB. This instrument along with others, is not an exception. Chong and Liu (2009) in their study found that there is no significance difference between conventional and Islamic systems. As Haider (2007) concluded, as an alternative system of commerce, Islamic finance has not been successful in achieving its articulated and functional goal in regards to the notions of fairness and social justice.

This study, thus, aims to construct mathematical formulations for AITAB contract which are in accordance to *Shari'ah* rules and regulations. The aim of this research paper is to propose the mathematical formulation of profit-sharing ratio (PSR) using stochastic programming which is deemed to suit the AITAB agreement better.

This article is divided into three major parts excluding this introduction. In the second section, the Islamic economy system and AITAB, the Islamic hire-purchase contract, is briefly introduced. The third section is the construction of PSR mathematical formulation and the determination of term charges vector is the subject of the fourth section.

### **The Foundation Concept of Islamic Banking & Finance and Islamic Hire-purchase**

Islam is a religion from God (Allah SWT) through His messenger Prophet Muhammad SAW, with the Holy Quran as central to Islamic beliefs (Kettell, 2008). The underlying legal principles of Islamic banking and finance are *Shari'ah*. *Shari'ah* is the Islamic law consisting of a set of rules and regulations pertaining to the relationship between humans and Allah SWT and humans with one another (Kettell, 2008). It covers all aspects of human life including banking and finance. There are four sources of *Shari'ah* with the Holy Quran as the main source, followed by the Sunnah or Hadith (actions or saying of Prophet Muhammad SAW), Ijma (consensus of opinion of the Muslim jurists) and lastly Qiyas (analogy of Muslim jurists) (Sudin, 1996).

### **Islamic Banking and Finance**

Islamic banking and finance system is very different from the conventional system because its foundation is based on *Shari'ah*.

Islamic banking and finance system has gone through three eras: (i) from the Prophet Muhammad SAW preaching of Islam in Mecca

until the Caliph ar-Rashidin (close friends of Prophet Muhammad SAW) periods (ii) caliphates era until the fall of Uthmaniyah Empire (Islamic Empire) (iii) modern Islamic banking and finance which is from the fall of Uthmaniyah Empire until now (Sudin, 2009). Recently Islamic banking and finance has stolen a lot of attention around the globe with over 300 banks operating using Islamic banking and financing concept (Amir and Ali, 2007).

It is imperative that every banking and finance system has inter-linkage of five components. They are: (i) money is the onset, (ii) financial instruments as a product of financial system, (iii) medium of transaction is the financial markets, (iv) financial institutions which offer the financial instruments, and (v) central bank to monitor the systems and promote stability of banking and finance (Abdul Ghafar, 2010). Referring to Abdul Ghafar (2010), Islamic system must embed four principles into the five components described above. The four principles are: (i) money as a capital and a transaction medium (ii) investor provides capital and entrepreneur deals with management (iii) financial transactions always involve risk sharing (iv) profit cannot be predetermined and should be determined by market forces (Abdul Ghafar, 2010).

Based on these principles, Islamic jurists list seven main methods for financing which is in line with the *Shari'ah* that can be devised by Islamic banks as listed in Table 1.

*Ijara Wa-Iqtina* or hire-purchase is one of the most widely-used method and is popular among customers. Hire-purchase contract is more preferable compared to other banking instruments

Table 1: *Shari'ah* Concepts.

| Method                                     | Description   |
|--|---|
| <i>Bai Bithaman Ajil</i>                   | Agreement for sale of goods with a deferred payment   |
| <i>Ijara orleasing</i>                     | Contract of leasing asset requested by client for rental fee  |
| <i>Ijara Wa-Iqtina</i><br>or hire-purchase | Agreement for leases out asset requested by client for rental fee with obligation for client to buy the asset at the end of the rental period |
| <i>Murabaha</i> or cost plus financing     | Sale contract of goods with profit margin agreed by contracting parties   |
| <i>Mudaraba</i> or profit-loss sharing     | Agreement of partnership with one side providing capital and the other side providing entrepreneurial skills                                  |
| <i>Musharaka</i> or partnership financing  | Partnership with contracting parties providing capital  |
| <i>Qard ul hasan</i> or benevolent loan    | Loan which the borrower only having to pay the principal  |

Source: Tlemsani et al. (2010)

because of its tax exemption (Mandell, 2002). Moreover, it gives an opportunity for those who cannot afford to buy goods on cash basis to use and own goods through periodic monthly payments (Seif I.Tag and N.Irwani, 2007). Hire-purchase is also the answer to Islamic sector which operates on interest-free principles to avoid interest rate risk (Radiah and Leong, 2009). Thus, Islamic hire-purchase is an important and powerful tool which is in line with *Shari'ah* that could be used and fully developed to utilise its benefits. In the following section, the Islamic hire-purchase contract used in Malaysia known as *Al-Ijara Thumma Al-Bai'* or AITAB will be described.

### ***Islamic Hire-Purchase Contract***

In Islam, *Ijara* or lease agreement has two different perspectives which are (i) a hire contract to employ services of a person, or (ii) a lease contract of particular asset (Usmani, 2006). In this study, interest is only in the second type of lease agreement.

There are two types of lease agreement in Islam namely *Ijara* (or lease) and *Ijara Wa-Iqtina* (or hire-purchase) as presented in Table 1. In this research, only hire-purchase contract is considered since it is the most widely used and attractive to banks and customers.

*Ijara Wa-Iqtina* is a contract which transfers the ownership of a leased asset to lessee in the form of sale. Referring to Bank Negara Malaysia (2009), AITAB is a form of *Ijara Wa-Iqtina* and the only Islamic hire-purchase contract in Malaysia. It was introduced in 1995 by Bank Islam (N.Irwani and Asyraf Wadji, 2006) and consists of two different agreements: *Al-Ijara*, which is a hiring agreement and *al-bai'*, which is a purchasing agreement. Hiring agreement will occur first followed by purchasing agreement. Therefore, the process of transferring hiring agreement into purchasing agreement without condition is an AITAB contract (Abdul Ghafar, 2010).

Consider two parties participating in the AITAB contract, namely the lessor and the lessee. The lessor is the owner of the leased asset while lessee gains the usufruct of the asset by renting for a certain time period agreed. Normally, the lease term will use up most of the leased asset's lifetime

span (Myres *et al.*, 1976), hence the lessor will usually pass ownership of the leased asset to the lessee at the maturity date of the AITAB contract. During the leased period, lessor obtains monthly rent which is equal to the periodic payments of the leased asset price (or periodic payments of principal) and a profit. On the other hand, lessee obtains the benefits from using the asset as well as ownership at the end of the contract. But if the lessee in any way defaults on the rent payment, the lessor has the right to repossess the asset.

Referring to studies done by previous researchers, among others Abdul Sattar (1998), Seif I.Tag and N. Irwani (2007), Mohammad Hashim (2007), Ala' Eddin (2004), N.Irwani and Asyraf Wadji (2006), Ros Aniza and Abdul Rahim (2003) and Usmani (2006), Islamic hire-purchase agreement need to follow specific rules and regulations as stipulated by the *Shari'ah* in order to make it valid in the Islamic perspective. Thus, in this study, the implementation of *Mudharaba* or profit-and-loss sharing (also known as partnership) and *Musyaraka Mutanaqisah* or diminishing ownership into AITAB modus operandi is done and explained.

### ***Modus Operandi of Al-Ijara Thumma Al-Bai' with Mudharaba (Profit and Loss Sharing) and Musyaraka Mutanaqisah (Diminishing Ownership)***

Consider an AITAB contract with a lessor and a lessee. At time  $t=0$ , the lessor must make a decision of whether or not to enter into the AITAB contract for  $T$  months with the lessee. It is assumed that the lessor will only agree to enter into the contract if and only if he believes the benefit from the AITAB transaction is at least equal to risk-free investment return for the same duration and principle. This is due to the fact that in a dual financial system, Islamic investment return, though it is not based on interest, must be as comparative to the conventional, therefore must be at least equal to the risk-free return in order to attract investors/customers. Thus, this is the first constraint to consider.

Furthermore, based on the Hire-Purchase Act 1967 (Act 212) (Legal Research Board, 2009), the lessee must give an advance of at least 10%

of leased asset price, with the balance borne by the lessor, before the agreement is to be bounded. Considering that, the lessee is also a partner to the lessor, at the same time the renter of the asset. Thus, the profit must be equally shared between the two partners. Therefore, the problem of determining the right profit sharing ratio between the lessor and the lessee is the main aim of this paper and will be discussed in the next section.

According to Abdul Aziz (1992), rent consists of periodic principal payments and compensation for benefits gained through the leased asset by lessee. Compensation for gains of the leased asset is acquired through the profit rate. Summing up, the periodic principal payments will be equal to the lessor's principal. Thenceforth lessor's ownership will decrease with time and will be zero at time interval  $t=\{T-1\}$ , and the contrary will happen to lessee's ownership. The concept applied here is known as *Musyaraka Mutanaqisah*. Based on Abdul Aziz (1992), modelling the *Musyaraka Munataqisah* (diminishing ownership) concept is embedded into our AITAB modelling.

Profit generated is based on the profit rate. As currently practised, the Islamic sector only considers a fixed profit rate of return, even though in reality it is contradictory. Fixed interest rates are not permitted in Islam (Karsten, 1982). Thus a fluctuating profit rate is considered which is assumed to be random and varies with time interval on monthly basis. Randomness in profit rate is permitted in *Shari'ah* (Islamic law) because it gives justice to the contracted parties since it reflects true market forces providing an opportunity for both parties to generate more benefits (Abdul Sattar, 1998). The discussion of steps taken in each time intervals of AITAB contract and the benefit equations is given in the next section.

**Mathematical Formulation of Profit-sharing Ratio Determination for Islamic Hire-purchase Contract**

*Al-Ijara Thumma Al-Bai' Benefit Equations*

The construction of mathematical models for the equations to show AITAB's benefit are depicted in the process as given below. Equations describe the event taking place at each time interval,  $t$ . First

it is necessary to understand the situations taking place within the AITAB contract at each time  $t$  with  $t = 0,1,\dots, T-1$ . The notations used in this paper are given as follows:

- $P$  : principal or lessor's investment
- $a_{t+1}$  : periodic principal payment at time interval  $t+1$
- $X_t$  : lessor's ownership at time interval  $t$
- $Y_t$  : lessee's ownership at time interval  $t$
- $\theta_t^{pp}$  : lessor's profit-sharing ratio at time interval  $t$
- $\theta_t^p$  : lessee's profit-sharing ratio at time interval  $t$
- $B_t^{pp}$  : lessor's benefit at time interval  $t$
- $B_t^p$  : lessee's benefit at time interval  $t$
- $u_{t+1}(\omega)$  : profit rate (or term charges) at time  $t+1$
- $r_f$  : risk-free rate

At the beginning of each time interval  $t$ , the participating parties must determine profit-sharing ratio before the realisation of profit rate (or terms charges). The terms charges are random variables that change in each month.  $B_t^{pp}$  and  $B_t^p$  are lessor's profit ratio and lessee's profit ratio accordingly. These arguments apply to all time intervals throughout the AITAB contract.

**At time  $t = 0$**

The lessor initial contribution is at most  $0.9P$  and lessee initial contribution is at least  $0.1P$ . Thus  $X_0 = (0.9P) - a_1$  and  $Y_0 = (0.1P) + a_1$ . Then  $X_0 - Y_0 = P$ .

Lessor's and lessee's benefits are given by the equations below.

$$B_0^{pp} = ((0.9P) - (a_1 u_1(\omega) \theta_0^{pp})) + ((0.1P) + (a_1 u_1(\omega) \theta_0^{pp}))$$

$$= (X_0 + Y_0) u_1(\omega) \theta_0^{pp} = P u_1(\omega) \theta_0^{pp}$$

$$B_0^p = ((0.9P) - (a_1 u_1(\omega) \theta_0^p)) + ((0.1P) + (a_1 u_1(\omega) \theta_0^p))$$

$$= (X_0 + Y_0) u_1(\omega) \theta_0^p = P u_1(\omega) \theta_0^p$$

**At time  $t = 1$**

$X_1 = X_0 - a_2$  and  $Y_1 = Y_0 + a_2$ . Then  $X_1 + Y_1 = X_0 + Y_0 = P$ .

$$B_1^{pp} = (X_0 - (a_2 u_2(\omega) \theta_1^{pp})) + (Y_0 + (a_2 u_2(\omega) \theta_1^{pp}))$$

$$= (X_1 + Y_1) u_2(\omega) \theta_1^{pp} = P u_2(\omega) \theta_1^{pp}$$

$$B_1^p = (X_0 - (a_2 u_2(\omega) \theta_1^p)) + (Y_0 + (a_2 u_2(\omega) \theta_1^p))$$

$$= (X_1 + Y_1) u_2(\omega) \theta_1^p = P u_2(\omega) \theta_1^p$$



At time  $t = 2$

$X_2 = X_1 - a_3$  and  $Y_2 = Y_1 + a_3$ . Then  $X_2 + Y_2 = X_1 + Y_1 = P$ .

$$B_2^{pp} = (X_1 - (a_3 u_3(\omega) \theta_2^{pp})) + (Y_1 + (a_3 u_3(\omega) \theta_2^{pp}))$$

$$= (X_2 + Y_2) u_3(\omega) \theta_2^{pp} = P u_3(\omega) \theta_2^{pp}$$

$$B_0^p = (X_1 - (a_3 u_3(\omega) \theta_2^p)) + (Y_1 + (a_3 u_3(\omega) \theta_2^p))$$

$$= (X_2 + Y_2) u_3(\omega) \theta_2^p = P u_3(\omega) \theta_2^p$$

At time  $t = T-1$

$X_{T-1} = X_{T-2} - a_T$  and  $Y_{T-1} = Y_{T-2} + a_T$ . Then  $X_{T-1} + Y_{T-1} = X_{T-2} + Y_{T-2} = P$ .

$$B_{T-1}^{pp} = (X_{T-2} - (a_T u_T(\omega) \theta_{T-1}^{pp})) + (Y_{T-2} + (a_T u_T(\omega) \theta_{T-1}^{pp}))$$

$$= (X_{T-1} + Y_{T-1}) u_T(\omega) \theta_{T-1}^{pp} = P u_T(\omega) \theta_{T-1}^{pp}$$

$$B_{T-1}^p = (X_{T-2} - (a_T u_T(\omega) \theta_{T-1}^p)) + (Y_{T-2} + (a_T u_T(\omega) \theta_{T-1}^p))$$

$$= (X_{T-1} + Y_{T-1}) u_T(\omega) \theta_{T-1}^p = P u_T(\omega) \theta_{T-1}^p$$

Based on the equation at each time interval above, it can be concluded that the total benefit for lessor and lessee is given by equation (1) and equation (2) respectively.

$$B^{pp} = \sum_0^{T-1} P u_{t+1}(\omega) \theta_t^{pp} \tag{1}$$

$$B^p = \sum_0^{T-1} P u_{t+1}(\omega) \theta_t^p \tag{2}$$

Referring to Usmani's (2006) proposal, profit-sharing ratio can be determined based on the ownership. Therefore the changing sharing ratio (NPB) is given by equation (3) and (4). With equation (3) is the lessor's NPB and equation (4) is the lessee's NPB.

$$\theta_t^{pp} = (X_t - a_{t+1}) P^{-1} \tag{3}$$

$$\theta_t^p = (Y_t - a_{t+1}) P^{-1} \tag{4}$$

It is obvious that  $\theta_t^{pp} + \theta_t^p = 1$  at all  $t$ .

As shown above, it is apparent that the benefit for lessor and lessee at each time interval depends on the profit rate and the profit-sharing ratio. The profit-sharing ratio must be decided at the beginning of each time period  $t$  before the realisation of profit rate  $u_{t+1}(\omega)$ . Equations (3) and (4) are deterministic sharing ratios and are based on the ownership known as the ratio-identity approach (Zubair, 1985). Considering the randomness within  $u_{t+1}(\omega)$ , the deterministic approach is deemed to be less suitable and the stochastic approach is more preferable (Kall and Mayer, 2005).

**Profit Sharing Ratio Modelling with Stochastic Programming**

To overcome the issue of indeterministic profit, the profit-sharing ratio (PSR) determination given by  $\vartheta_t$  is introduced. Using stochastic programming approach, a model for the PSR determination is constructed.

In this paper, only the lessor's PSR is considered, given that  $\vartheta_t^{pp} + \vartheta_t^p = 1$ , given that  $\vartheta_t^{pp}$  and  $\vartheta_t^p$  is lessor's PSR and lessee's PSR accordingly. The assumptions considered in the modelling are as follow:

- (A1) Lessor's profit from AITAB contract must be at least equal to the risk-free return.
- (A2) Malaysian government bond is the risk-free security considered here with fixed interest rate and  $T$  maturity duration.
- (A3) Terms charges (or profit rate) considered is changing within bounded set given by Base Financing Rate (BFR) with the maximum BFR is and the minimum is.
- (A4) The tax exemption covers the depreciation of asset.
- (A5) Lessee is indifferent towards profit.

Since we assumed lessee is indifferent towards profit and for simplicity of modelling, only the lessor side is considered in this mathematical modelling. The modelling is done in lessee's perspective taking into consideration that the maintenance of leased asset is borne by lessee. Therefore, the optimisation problem is to minimise the lessor's profit-sharing ratio with certain constraints as given in equation (5).

$$\min \vartheta_t^{pp} \tag{5}$$

s.t:

$$\vartheta_t^{pp} \geq \frac{(X_0 - ta)r_f}{u_{t+1}(\omega) P} ; \forall t$$

$$\vartheta_t^{pp}, u_{t+1}(\omega), r_f \geq 0; \forall t$$

given that for all  $t \in [0, T]$  (in month):

$$X_0 \leq 0.9 \times P$$

$$a = \frac{0.9P}{T}, \text{ and}$$

$$\left[ \frac{(X_0)(\omega_{12}^-)}{100\{1 - (1 + [\omega_{12}^- \times 100]^{-T})\}} \times T \right] - (X_0) \leq u_{t+1}(\omega)$$

$$\leq \left[ \frac{(X_0)(\omega_{12}^+)}{100\{1 - (1 + [\omega_{12}^+ \times 100]^{-T})\}} \times T \right] - (X_0)$$

With  $\vartheta_t^{pp}$ ,  $\omega_{12}^+$  and  $\omega_{12}^-$  representing the lessor's profit-sharing ratio, upper BFR and lower BFR respectively. The terms charges,  $u_{t+1}(\omega)$  is a random variable varying monthly depending on BFR,  $\omega$ .  $\omega$  is given by  $\omega^- \leq \omega \leq \omega^+$ , with  $\omega^+ = 10.75\%$ ; this value is stated by the Central Bank of Malaysia (Bank Negara Malaysia, 1994) and  $\omega$  is assumed to be the lowest BFR in the historical data.  $r_f, P$  and  $a$  are constants.

From equation (5), the optimal minimum value for lessor's PSR,  $\vartheta_t^{pp}$  is obtained. The constraint in equation (5) is constructed based on the assumption that lessor's profit must be at least equal to the risk-free return. This is given by equation (6) below.

$$P\vartheta_t^{pp}u_{t+1}(\omega) \geq X_t r_f \quad (6)$$

with  $X_t = X_0 - ta$ .

The risk-free security considered is the Malaysia government bond namely Sekuriti Kerajaan Malaysia (SKM). These securities are considered to be risk-free because the likelihood of government defaulting is almost nil. Furthermore the short or medium maturities protect investor from interest-rates risk. The interest rates for these securities are fixed throughout the entire durations.

The optimisation problem designed here is an uncertain linear problem due to the randomness in term charges. Thus stochastic programming is used to obtain the optimal solution for the problem by finding the highest probability of an event occurring given it must fulfil all constraints. Therefore, the equation is given by (8).

$$\left\{ \min \left\{ \vartheta_t^{pp}; \mathbb{P} \left\{ u_{t+1}(\omega); \vartheta_t^{pp} \geq \frac{(X_0 - ta)r_f}{u_{t+1}(\omega)P} \right\} \geq \alpha \right\}; u_{t+1}(\omega) \in \mathbb{U} \right\} \quad (8)$$

with  $\mathbb{U}$  is the uncertainty set and the optimal solution for equation (8) is the lessor's optimum PSR.

### The Term Charges Determination

Referring to the discussion in subsection Profit Sharing Ratio Modelling with Stochastic Programming, it is stated that the terms charges,  $u_{t+1}(\omega)$  is a random variable varying monthly depending on BFR,  $\omega$ .  $\omega$  is given by  $\omega^- \leq \omega \leq \omega^+$  and is also a random variable. In fact, the randomness within  $u_{t+1}(\omega)$  occur because it depends on  $\omega$ . Therefore the distribution of  $\omega$  must be first determined.

### Base Financing Rate Distribution

The banking and financing system in Malaysia is unique with conventional system and Islamic system working alongside each other. However, the Islamic system differs from conventional system in various ways. The main difference is that Islamic system does not operate on interest basis. Due to this fact, BNM has introduced the Base Financing Rate (BFR) for Islamic system to differentiate from the Base Lending Rate (BLR) used by conventional system.

BFR and BLR determination do not differ much. Due to limited historical data, it is reasonable to consider BLR as a liaison unit. This consideration is backed by the strong positive correlation result as exhibited in Table 2. This finding is in accordance with BNM report indicating that there exists a relationship between BFR and BLR (Bank Negara Malaysia, 2006).

Table 2: Correlation of Base Financing Rate (BFR) and Base Lending Rate (BLR)

| Control Variables |     |                         | BFR   | BLR   |
|-------------------|-----|-------------------------|-------|-------|
| Month             | BFR | Correlation             | 1.000 | .998  |
|                   |     | Significance (2-tailed) | .     | .000  |
|                   |     | df                      | 0     | 14    |
| BLR               | BLR | Correlation             | .998  | 1.000 |
|                   |     | Significance (2-tailed) | .000  | .     |
|                   |     | df                      | 14    | 0     |

A distribution test on the BLR monthly data was therefore conducted starting from January 1998 until May 2010. The results show that BLR follows a log logistic distribution with the highest correlation coefficient, 0.858 as shown in Figure 1.

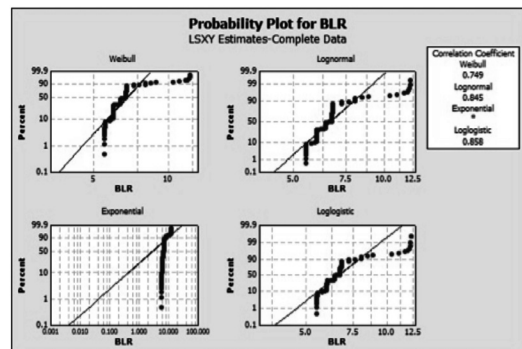


Figure 1: Base Lending Rate (BLR) distribution

**Term Charges Vector**

Further, let it be assumed there exists a random set  $\Omega \mathfrak{S}(\Omega)$  with power . For every set of  $(A_i)_{i \in I}$  the following holds (Bauer (1981), Oksendal (1998)):

- (i)  $(A_i)_{i \in I} \subset \mathfrak{S}(\Omega)$
- (ii)  $\cup (A_i)_{i \in I} \subset \mathfrak{S}(\Omega)$
- (iii)  $\cap (A_i)_{i \in I} \subset \mathfrak{S}(\Omega)$
- (iv) for every set  $A, (A)^c \subset \mathfrak{S}(\Omega)$

Then by definition, a system of sets  $\mathcal{F}$  is  $\sigma$ -algebra in  $\Omega$  iff:

- (i)  $\mathcal{F} \subset \mathfrak{S}(\Omega)$
- (ii)  $\Omega \in \mathcal{F}$
- (iii)  $A \in \mathcal{F} \Rightarrow (A)^c \in \mathcal{F}$
- (iv) for every sequence  $(A_n)$  of sets of  $\mathcal{F}, \cup (A_n)$  with  $n = 1, 2, \dots$  lies in  $\mathcal{F}$ .

Let  $\omega_t$  be a random variable (as shown in section Base Financing Rate Distribution). Then it is defined that  $\mathcal{F}_t$  is  $\sigma$ -algebra generated by random variables  $\{\omega_s\}_{0 \leq s \leq t}$ . Thus,  $\mathcal{F}_t$  is the historical  $\omega_s$  to  $t$ . Having this, the term charges,  $u_{t+1}(\omega)$  which depends on  $\omega$  are also random and can be determined based on  $\mathcal{F}_t$ .

But,  $\omega$  cannot be negative and have an upper limit. Therefore assumption is made that it is in a set of  $[0, \varepsilon^+]$ . Thus the following situation is considered.

**Situation 1**

Consider  $0 \leq u_{t+1}(\omega) \leq \varepsilon^+$  given  $\vartheta_t^{pp} \geq \frac{c}{\dot{u}_{t+1}(\omega)}; \forall \omega$ .

If the lower bound limit of  $u_{t+1}(\omega) \rightarrow 0$ , then  $\vartheta_t^{pp} \geq \frac{c}{\dot{u}_{t+1}(\omega)} = +\infty = \emptyset$ . Thus, it is not bounded.

Meanwhile, if the upper bound limit of  $u_{t+1}(\omega) \rightarrow \varepsilon^+$ , then  $\vartheta_t^{pp} \geq \frac{c}{\dot{u}_{t+1}(\omega)} = \sigma^+$  which is bounded.

**Situation 2**

Consider  $\varepsilon^- \leq u_{t+1}(\omega) \leq \varepsilon^+$  given  $\vartheta_t^{pp} \geq \frac{c}{\dot{u}_{t+1}(\omega)}; \forall \omega$ .

If the lower bound limit of  $u_{t+1}(\omega) \rightarrow \varepsilon^-$ , then  $\vartheta_t^{pp} \geq \frac{c}{\dot{u}_{t+1}(\omega)} = \sigma^+$  thus it is bounded.

Meanwhile if the upper bound limit of  $u_{t+1}(\omega) \rightarrow \varepsilon^+$ , then  $\vartheta_t^{pp} \geq \frac{c}{\dot{u}_{t+1}(\omega)} = \sigma^+$  which is also bounded.

From situation 1, it is obvious that there is no bounds for  $\vartheta_t^{pp} \geq \frac{c}{\dot{u}_{t+1}(\omega)}; \forall \omega$  if the  $u_{t+1}(\omega)$  is

approaching to zero. Then it makes sense to limit the lower bound as in situation 2 with  $\varepsilon^- \leq u_{t+1}(\omega) \leq \varepsilon^+$  given given  $\omega^- \leq \omega \leq \omega^+$ .

**Conclusion**

The Islamic hire-purchase model proposed is structured by the *Mudharaba* and *Musyaraka Mutanaqisah* principles embedded in the profit-sharing ratio determination between the contracting parties. The profit of the contract is determined by the profit rate which is fluctuates. A fluctuating profit rate complicates the process of determining the profit-sharing ratio since it is not deterministic. Thus a stochastic programme model is constructed to determine the optimal PSR.

The proposed PSR model captures the randomness of the benefits gained as a result of the fluctuating profit rate. The constraint involved is that benefit of the lessor must be at least equal to the risk-free investment return. Thus it is crucial to obtain a sufficient PSR for the lessor at the beginning of each time interval in order to meet the constraint. Hence this will result in a more efficient and reliable process.

**Acknowledgements**

We would like to acknowledge Professor Dr. Rdiger Schultz from Fakultt fr Mathematik, Universitt Duisburg-Essen, Germany for his valuable advice, comments and suggestions, regarding the mathematical aspects of this paper. We would also like to thank the helpful comments from the editor and reviewers.

**References**

Abdul Aziz, J. (1992). Perbandingan Pembayaran Pinjaman Cara Islam dan Konvensional. *MATEMATIKA*. 8(2): 133-142.

Abdul Ghafar, I. (2010). *Money, Islamic Banks and the Real Economy*. Cengage Learning.

Abdul Sattar, A. G. (1998). *Ijara* (Lease). Technical report, Al-Baraka Banking Group.

Amir, K., & Ali, F. D. (2007). Modeling money demand under the profit-sharing banking scheme: Some evidence on policy invariance and long-run stability. *Global Finance Journal*. 18: 104-123.

- Bank Negara Malaysia (1994). *Wang dan Urusan Bank di Malaysia*. Bank Negara Malaysia.
- Bank Negara Malaysia (2006). *Annual Report 2006*. Bank Negara Malaysia.
- Bank Negara Malaysia (2009). Draft of *Shari'ah* parameter reference 2: Ijarah contract. Technical report, Bank Negara Malaysia.
- Bauer, H. (1981). *Probability Theory and Elements of Measure Theory*. Academic Press.
- Chong, B. S., & Liu, M. H. (2009). Islamic Banking: Interest-Free or Interest- Based? *Pasific-Basin Finance Journal*. 17: 125-144.
- Haider, A. H. (2007). Jurisprudential Schizophrenia: On Form and Function in Islamic Finance. *Legal Studies Reserach Paper Series*. (2007-10).
- Kall, P., & Mayer, J. (2005). *Stochastic Linear Programming: Models, Theory, and Computation*. Springer.
- Karsten, J. (1982). Efforts to develop Islamic financial systems: focus on the abolition of fixed interest rates. *OPEC Bulletin*, 13(3).
- Kettell, B. (2008). *Introduction to Islamic Banking & Finance*. Pinthaus.
- Legal Reserach Board (2009). *Hire-Purchase Act 1967 (Act 212) & Regulations*. International Law Book Services.
- Mandell, S. (2002). Lessor and Lessee Perspectives on Ground Lease Pricing. *Journal of Property Research*. 19(2): 145-157.
- Muhammad, A. (2007). *Understanding Islamic Finance*. John Wiley & Sons.
- Myres, S. C., Dill, D. A., & Bautista, A. J. (1976). Valuation of financial lease contract. *Journal of Finance*. 31(3): 799-819.
- Irwani, N. A., & Asyraf Wadji, D. (2006). A Critical Appraisal of *Al-Ijara Thumma Al-Bay'* (AITAB) Operation: Issues and Prospects. *4<sup>th</sup> International Islamic Banking and Finance*.
- Oksendal, B. (1998). *Stochastic Differential Equations*. Springer.
- Radiah, A. K., & Leong, Y. K. (2009). The Impact of Interest Rate Changes on Islamic Bank Financing. *International Review of Business Research Papers*. 5(3): 189-201.
- Seif, I., Tag, E. D., & N. Irwani, A. (2007). Issue of Implementing Islamic Hire-Purchase in Dual Banking Systems: Malaysia's Experience. *Thunder-bird International Business Review*. 49(2): 225-249.
- Sudin, Haron (1996). Islamic Banking: A New Vehicle In Fostering Entrepreneurship. *Journal of Islamic Banking and Finance*. 13(3): 28-39.
- Sudin, Haron (2009). *Islamic Finance and Banking System Philosophies, Principles & Practices*. Mc Graw Hill.
- Tlemsani, I., & Matthews, R. (2010). *Ethical Banking: The Islamic View*. Kingston Business School.
- Usmani, M. T. (2006). *Ijarah*. Retrieved from <http://www.accountancy.com.pk>
- Zubair, H. (1985). Determination of Profit and Loss Sharing Ratio in Interest-Free Business Finance. *J. Res. Islamic Econ*. 3(1): 13-29.