

**THE DEVELOPMENT OF PLANAR CURVES WITH
HIGH AESTHETIC VALUE**

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THE DEVELOPMENT OF PLANAR CURVES WITH HIGH AESTHETIC VALUE

by

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CS101	Computer Architecture
CS102	Computer-Aided Aesthetic Design
CS103	Computer-Aided Design
CS104	Computer-Aided Design & Computer-Aided Manufacturing
CS105	Computer-Aided Geometric Design
CS106	Computer-Aided Manufacturing
CS107	Computer-Aided Styling
CS108	Computer Graphics
CS109	Computer Numerical Control
CS110	Digital Masking
CS111	Formalization and Integration of an Optimized Process
CS112	Engineering Spring Workflow
CS113	Generalized Conic Spirals
CS114	Generalized Log-Rectified Curves
CS115	Involute From Circular Arc
CS116	Involute From Straight Line
CS117	Log-Rectified Curve
CS118	Logarithmic Curvature Graphs
CS119	Logarithmic Distribution Diagram of Curvature
CS120	Logarithmic Distribution Graph of Curvature
CS121	Linear Curvature Element
CS122	Minimum Curvature Variation
CS123	Minimum Variation Curves
CS124	North American Arabesque
CS125	Optimized Geometric Heuristics Curves
CS126	Scholarly Computation Program
CS127	Standard Translation Language
CS128	Total Least Square

LIST OF ABBREVIATIONS

Abbreviation	Description
BLINCE	Bilinear Curvature Element
CAAD	Computer Aided Aesthetic Design
CAD	Computer Aided Design
CADCAM	Computer Aided Design & Computer Aided Manufacturing
CAGD	Computer Aided Geometric Design
CAM	Computer Aided Manufacturing
CAS	Computer Aided Styling
CG	Computer Graphics
CNC	Computer Numerical Control
DMU	Digital Mockup
FIORES	Formalization and Integration of an Optimized Reverse Engineering Styling Workflow
GCS	Generalized Cornu Spiral
GLAC	Generalized Log-Aesthetic Curve
IFCA	Involute From Circular Arc
IFSL	Involute From Straight Line
LAC	Log-Aesthetic Curve
LCG	Logarithmic Curvature Graph
LDDC	Logarithmic Distribution Diagram of Curvature
LDGC	Logarithmic Distribution Graph of Curvature
LINCE	Linear Curvature Element
MCV	Minimum Curvature Variation
MVC	Minimum Variation Curves
NAA	North American Aviation
OGH	Optimized Geometric Hermite Curves
SCP	Scientific Computation Program
STL	Standard Tessellation Language
TLS	Total Least Square

PEMBANGUNAN LENGKUNG SATAH DENGAN NILAI ESTETIK YANG TINGGI

ABSTRAK

Penyelidikan terhadap lengkung satah bagi menghasilkan suatu produk yang cantik dan pengubahsuaian lengkung bagi bidang tertentu telah dibangunkan sejak tahun 70-an. Pola penyelidikan dalam bidang ini boleh dibahagikan kepada lima cabang utama iaitu pembangunan algoritma adil, pembangunan lengkung satah melalui kaedah sintesis lengkung, pembangunan algoritma baru bagi mengubahsuai lingkaran asli untuk kegunaan reka bentuk, pengubahsuaian lengkung fleksibel (Bézier dan NURBS) supaya profil kelengkungan adalah monoton dan akhirnya, pembangunan algoritma dalam proses penyuaian dan penghampiran terhadap lingkaran asli menerusi lengkung fleksibel. Bulatan involut adalah lengkung satah yang mempunyai nilai estetik tinggi dan terkenal untuk reka bentuk gerigi gear. Sumbangan pertama adalah pengubahsuaian bulatan involut agar dapat disesuaikan dalam bidang reka bentuk. Proses evolut-involut digunakan bagi menjana dua jenis splin; involut dari garis lurus (IFSL) dan involut dari lengkok bulatan (IFCA). Sumbangan kedua penyelidikan ini adalah pengenalan dan penambahbaikan Graf Kelengkungan Logaritma (LCG) untuk menghasilkan lengkung yang mempunyai nilai estetik yang tinggi. Persamaan LCG dan kecerunan LCG digunakan sepanjang kajian yang dijalankan ini sebagai suatu kaedah siatan reka bentuk efektif. Analisis terhadap lingkaran Cornu teritlak (GCS) telah menjelaskan elemen yang tepat bagi mengenalpasti suatu lengkung satah mempunyai nilai estetik. Oleh kerana GCS dikenali sebagai suatu lengkung estetik, penelitian dilakukan terhadap pembentukan lengkung baru yang mempunyai kecerunan LCG sebagai suatu persamaan garis lurus. Kajian ini telah menghasilkan lengkung log estetik teritlak (GLAC). Lengkung ini mempunyai darjah kebebasan tambahan dan terdiri dari pelbagai lengkung seperti Nielsen spiral, Logarithma spiral, clothoid, bulatan involut, segmen Lengkung Log Estetik (LAC) dan GCS. Akhirnya, pengukuran berangka bagi nilai estetik lengkung satah dilakukan dengan mengubahsuai formula Birkhoff. Hasil kajian membuktikan bahawa formula tersebut boleh digunakan setelah clothoid mendapat ukuran nilai estetik tertinggi berbanding dengan lengkung satah yang lain.

THE DEVELOPMENT OF PLANAR CURVES WITH HIGH AESTHETIC VALUE

ABSTRACT

The research on developing planar curves to produce visually pleasing products and modifying planar curves for special purposes has been progressing since the 1970s. The pattern of research in this field of study has branched to five major groups; the development of fairing algorithms; the development of planar curves via curve synthesis, the development of algorithms to modify natural spirals to suit design intent, the modification of flexible curves (Bézier and NURBS) so that the curvature profile is strictly monotonic and finally, the development of algorithms to fit natural spirals and approximation via flexible curves. A circle involute is a planar curve with high aesthetic value and it is famous for gear teeth design. The first contribution is the algorithm to construct circle involute to suit curve styling/design environment. Using the evolute-involute process, two types of splines were developed; involute from straight lines (IFSL) and involutes from circular arcs (IFCA). The second contribution of this research is the identification and enhancement of Logarithmic Curvature Graph (LCG) in order to identify and develop aesthetic curves. The LCG and its gradient formula have been used throughout this research as an effective shape interrogation tool. The analysis of GCS yielded an insight into what makes a curve aesthetic, where the ambiguity of a planar curve being aesthetic is elucidated. Since the GCS is identified as a potential aesthetic curve, the conditions for the curve to possess the LCG gradient as a straight line equation are identified. The extension of the investigation led to the Generalized Log Aesthetic Curve segment (GLAC). This planar curve has extra degrees of freedom and it comprises of many curves; Nielsen's spiral, Logarithmic spirals, clothoids, circle involutes, LAC segments and GCS segments. The methods to formulate GLAC segment are detailed. The final contribution is the numerical measurement formula to evaluate the aesthetic value of planar curves via the customization of Birkhoff's formula. The usability of the modified formula is valid since the clothoid scored the highest aesthetic value as compared to other planar curves.