STABILITY INDEX FOR RIDDLED BASINS OF ATTRACTION WITH APPLICATIONS TO SKEW PRODUCT SYSTEMS

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

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Abstract

This thesis examines how novel invariants called the "stability index" as proposed by Podvigina and Ashwin can be used to characterize the local geometry of riddled basins of attraction for both skew and non-skew product systems. In particular, it would be interesting to understand how the stability index behaves on the basin boundary between multiple basins of attraction. Then we can ask this question: How can we identify when a basin is riddled? To answer this, we present three models with the presence of riddled basins.

In the first model, we present a skew product system of a simple example of a piecewise linear map. We prove that the riddled basin occurs within a certain range of parameter and calculate the stability index analytically for this map. Our results for the stability index at a point show that for Lebesgue almost all points in the map, the index is positive and for some points the index may be negative. We verify these results with our numerical computation for this index. We also make a corollary claiming that the formula for the stability index at a point can be expressed in terms of the stability index for an attractor and Lyapunov exponents for this map. This suggests that this index could be useful as a diagnostic tool to study bifurcation of the riddled basins of attraction.

In the second model, we refer to a skew product map studied by Keller. Previously, Keller computed the stability index for an attractor in his map whereas in this thesis, we use an alternative way to compute the index; that is on the basins of attraction for Keller's map, found by inverting his map. Using the same map, we also verify maximum and minimum measures as obtained in his paper by studying Birkhoff averages on periodic points of Markov map in his system. We also conjecture result by Keller and Otani on the dimension of zero sets of invariant graph (i.e. basin boundary) that appears in Keller's map to a complete range of a parameter in the map.

The last model is a non-skew product map which is also has a riddled basin. For this map, we compute the stability index for an attractor on the baseline of the map. The result indicates that the index is positive for Lebesgue almost all points whenever the riddled basin occurs.