CONVOLUTION PRODUCT METHODS FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS

HASSAN ELTAYEB GADAIN

DOCTOR OF PHILOSOPHY OF MATHEMATICS
UNIVERSITI MALAYSIA TERENGGANU

2008

CONVOLUTION PRODUCT METHODS FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS

HASSAN ELTAYEB GADAIN

Thesis Submitted in Fulfillment of the Requirement for the Degree of Doctor of Philosophy of Mathematics in the Faculty of Science and Technology Universiti Malaysia Terengganu,

March 2008

Dedication

Specially Dedicated to

My wife, Afraa

And

My sons Mohamed, Monteser and Mozamel, My Daughters, Monia and Malaz

Abstract of thesis presented to the Senate of Universiti Malaysia Terengganu in fulfillment of the requirement for the degree of Doctor of Philosophy

CONVOLUTION PRODUCT METHODS FOR SOLVING PARTIAL DIFFERENTIAL EQUATIONS

HASSAN ELTAYEB GADAIN SALIM

March 2008

Chairman: Prof. Adem Kıhçman, Ph.D.

Member : Abd Fatah bin Wahab, Ph.D.
Mohd Lazim bin Abdullah, Ph.D.

Faculty : Science and Technology

In this thesis, the double convolutions properties are studied. We solved first order non-homogenous Partial Differential Equations (PDE) having non-homogenous initial condition by using reduction to first-order PDE technique. The Green's formula, Green's function and generalized function and the reduction to first-order PDE techniques have been used to solve the non-homogenous wave equation with initial condition. The non-homogenous terms used in this study are convolution terms. Furthermore the uniqueness and continuity of non-homogenous wave equation discussed.

Integral transform are extensively used in solving several kinds of boundary value problems and integral equations, since Laplace Transform is a special kind of integral transform can also be used to solve partial differential equation.

The linear second order PDEs with constant coefficients and non constant coefficients are solved by using Double Laplace Transform (DLT). We introduced Theorems and remainder function from the solutions, where the remainder appears at double convolution between the constant and non constant coefficients solutions. Here, the non constant coefficient is considered as convolutional product of polynomials. In special cases: we classified the non constant coefficients hyperbolic and elliptic equations after multiplying them by polynomials with convolution product. We obtained the same classifications as for the original equation. We noticed that the convolution product with polynomials coefficient keeps the classifications of the hyperbolic and elliptic equations. Double convolutions in Mellin sense and PDE with non constant coefficient are also studied using Double Mellin Transform. We note that Mellin Transform can be utilized to solve PDE containing non constant coefficients without boundary conditions.

Furthermore, we applied DLT to solve three fundamental equations. Non-homogenous terms of Wave, Poisson's and Heat equations are replaced by summation of double convolution functions and the non-homogenous data are replaced by a single convolution. The same technique is also used to solve the three fundamental equations having non constant coefficients. The non constant coefficients employed are polynomials or Trigonometric functions.

Finally, the concepts of generalized second order partial derivatives in two dimensions are given. In the second part, we extended the pseudofunction to two dimensions.